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PROFESSIONAL PAPERS

OF THE

CORPS OF ROYAL ENGINEERS.

(FOURTH SERIES).

EDITED BY MAJOR W. A. HARRISON,

SECRETARY, R.E. INSTITUTE.

PAPER 5.

STIFFENED SUSPENSION BRIDGES.

BY

MAJOR H. BIDDULPH, R.E.

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PREFACE.

COMMON stiffened suspension bridges are often met with in the more distant parts of the British Empire, but the subject is usually omitted in the ordinary book on construction; doubtless on account of the fact that such bridges are usually semi-permanent structures. In some standard works the method of investigation is only indicated, while in some other minor works either only one case is dealt with or the subject is treated erroneously in one way or another.

In the following pages an attempt has been made to make a comparison of the stresses in a suspension bridge stiffened by road-girders in three different ways, under exactly similar conditions, and with no alteration in any of the dimensions. The method of investigation in Chapters II. and III. is that indicated by Professor Claxton Fidler in his *Treatise on Bridge Construction*, while the elaboration of that method in Chapters II. and VI. is based on the work of Dr. Ritter.

STIFFENED SUSPENSION BRIDGES.

CHAPTER I.

INTRODUCTORY.

THE advantages of a field suspension bridge are well known; in economy of material, labour, and of time to construct it is on numerous occasions unsurpassed. It has however one serious defect, which is also well known, viz.:—Lack of rigidity, and liability to serious deformation under a moving, and therefore unsymmetrical load; and the higher the ratio of the moving load to the permanent load, the greater is this defect. Attempts are often made, especially in suspension bridges of a semi-permanent character, to minimize this evil, and to effect a compromise between a rigid braced structure, and the simple suspension bridge, by introducing a pair of stiffening girders, which carry the roadway, and are themselves suspended from the main cables.

It is obvious that, if they are of sufficient rigidity to prevent any appreciable distortion of the parabola of the main cables under a passing load, any unsymmetrical load is equally distributed by their means on to the cables, while they themselves are subjected to rather complicated stresses. In the following chapters three cases are investigated, with the same conditions of span, loading, dip, depth of girder, dimensions and wind pressure; and the results are tabulated for comparison in Chapter V.

First Case.—The stiffening girders are continuous over the span, resting on and anchored at the abutments, but capable of expanding and contracting longitudinally.

Here plainly the deflection of the girders, and the deflection of the cables, whether caused by loading or by temperature, must be the same; and as the deflection of the cables caused by any given alteration in temperature is constant, it follows that the consequent stresses caused in the girders vary with the depth of the girder, and that the greater the ratio of the depth of the girder to the dip of the cables, the greater the resulting temperature stress, and the less capable are the girders of dealing with any external load, and performing their proper function. On the other hand the smaller

their depth, the less rigid do they become, and in consequence less efficient and less able to prevent distortion of the main cables.

In this class of stiffening girder therefore the ruling factor is temperature stress; only a certain portion of the strength of the girder can be wasted in this unproductive manner, and having determined how much this is, the depth and dimensions of the girders can be ascertained.

It is obvious that the lengths of the vertical suspension rods could be so adjusted (after erection if necessary) so as to equalize the maximum hogging (positive) and the sagging (negative) bending moments in the girders.

It would be inadvisable however to endeavour to adjust the vertical suspension rods, so that the girders assumed a refinely calculated curve, and all that would be done in practice would be to relieve the girders of all the permanent load, which would be the case if the suspenders were adjusted so that the girders lay truly horizontal at the mean temperature. This has been allowed for as a correction to the calculated stresses at the end of Chapter II.

Second Case.—Here we have the girders resting as before on the abutments and anchored to them, but there is this radical alteration, that hinges are introduced at their centres. Clearly all temperature stresses are now eliminated automatically; the rise or fall of the cables cannot induce bending moments in the girders, nor can any permanent load do so, for the vertical suspension rods would be adjusted at the time of erection so that each half girder takes its proper alignment without stress. Consequently only the moving load can affect the girders, and as the stresses in them are independent of the deflection in the main cables, we can make the girders as deep as we like, and therefore the roadway as stiff as we like; the girders distributing equally on to the cables any moving load. The economy of this apparently small detail in the construction of the girders will be seen to be very great.

If the central hinges are so constructed as to permit of lateral movement as well, then no bending moment can be set up in the girders by lateral wind pressure, a still further advantage and economy, as otherwise this bending moment cannot be neglected.

Third Case.—In this instance we have continuous girders as in the first case, but instead of resting on the abutments they hang entirely suspended from the main cables.

It will be seen that under these conditions it is impossible to prevent distortion of the cables, and consequently the system is a faulty one *ab initio;* while a moving load must create a tendency of the bridge to oscillate along the direction of the roadway. As temperature stresses are not eliminated by this arrangement the maximum depth of the girder is limited, as in Case I., while if the roadway is very light compared with the moving load, it might actually lift at the ends

under certain conditions, owing to the great distortion of the cables, and the absence of anchorage bolts at the ends of the girders.

For purposes of comparison the data in all three cases are the same, viz. :—Span, 200 ft. ; dip, 200 in. ; roadway, 15 ft. wide ; moving load, 300 lbs. per ft.-run ; permanent load, 240 lbs. ; and wind load, 60 lbs. per ft.-run. Total cross-sectional area of main cables, 8 square inches, and that of the wind cables 1 square inch each. Total cross-sectional area of girder flanges, 16 square inches, with square, cross-braced panels, $50'' \times 50''$.

The wind girder is formed by the bottom flanges of the stiffening girders and cross-bracing beneath the roadway ; the panels being cross-braced, and square, $15' \times 15'$.

The " dip " of the wind cable is 300 in.

Maximum range of temperature $= \pm 50^{\circ}$ F.

Slope of back guys about 5 over 6.

The following points should be noted ;—

(1). The same value of E has been taken for the cables and the girders. This is true if the cables are of steel links, but not if of stranded steel rope. In the latter case the value of E for stranded steel rope would be less, and the necessary correction would have to be applied to the calculations for Cases I. and III. This reveals an additional advantage which the hinged girders have over the other two cases.

(2). In Cases I. and III., only *vertical* displacements of the cables are dealt with, and any horizontal displacements are neglected. The effect of doing so is an error on the side of safety, as any minute horizontal displacement would reduce the deflection of the cables.

(3). The neglect of the effect of temperature changes in the vertical suspension rods is also an error on the side of safety, for the standards undergo temperature changes which act in the opposite direction to the rise or fall of the cables, and tend to reduce it.

(4). The results attained by the calculations are not absolutely exact ; for the assumptions in Cases I. and II., that *every* unsymmetrical load is transferred as a uniform load to the cables, and that the curve of the cables is *always* a true parabola, are not absolutely true.

Chapter V. deals with the case of unbraced trussed beams, which will be seen to be merely a special case of a stiffened suspension bridge ; and the results demonstrate the lack of economy that results from the use of composite structures.

CHAPTER II.

SUSPENSION BRIDGE WITH CONTINUOUS GIRDERS.

LET *Fig.* 1 represent the cables of the suspension bridge, $2l$ *being the span*, and h the dip.

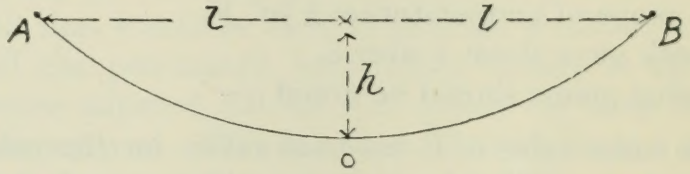

FIG. 1.

With O as origin, the equation to this parabola is

$$\frac{y}{h}=\frac{x^2}{l^2}.$$

$$\therefore \qquad \frac{dy}{dx}=\frac{2hx}{l^2},$$

and $$ds=dx\sqrt{1+\frac{4h^2x^2}{l^4}}=dx\left(1+\frac{2h^2x^2}{l^4}\right) \text{ approx.},$$

since we can neglect $-\frac{2h^4x^4}{l^8}$ and all succeeding terms.

Now if S be the length of the arc,

$$S=\int_{-l}^{l}\left(1+\frac{2h^2x^2}{l^4}\right)dx=2l\left(1+\frac{2h^2}{3l^2}\right).$$

Let P be the stress per square inch in the cables, and E the modulus of elasticity of their material; then λ, or the elongation per unit length of the cables, will equal $\frac{P}{E}$; and the length of the cables will become $S(1+\lambda)$ with a corresponding dip of h_1.

$$\therefore \qquad S(1+\lambda)=2l\left(1+\frac{2h_1^2}{3l^2}\right)=2l\left(1+\frac{2h^2}{3l^2}\right)(1+\lambda).$$

$$\therefore \qquad h_1=h\left\{1+\frac{\lambda}{2}\left(1+\frac{3l^2}{2h^2}\right)-\ldots\ldots\ldots\right\}.$$

The deflection of the cables

$$= h_1 - h = \frac{3\lambda l^2}{4h}\left(1 + \frac{2h^2}{3l^2}\right) \text{ approx.},$$

omitting the negligible terms of the series.

The fraction $\frac{2h^2}{3l^2}$ also is so small compared with unity, that it may be neglected without appreciable error, and we obtain

$$\frac{3\lambda l^2}{4h} \text{ as the deflection of the cables at the centre} \ldots\ldots [A].$$

The actual deflection will be greater than this, on account of the elongation of the back guys, which must remain taut always. It will be necessary therefore to know the proportion that the length of the back guys bears to the span; and for this example it will be assumed that their length equals one-sixth of the span, or $\frac{l}{3}$.

The length of the back guys at both abutments therefore equals $\frac{2l}{3}$, and their elongation will be $\frac{2\lambda l}{3}$ (assuming that their cross-section is proportional to the stress); therefore the deflection of the cables will be increased by $\frac{\frac{2l}{3}}{2l} = \frac{1}{3}$, *i.e.*, by $\frac{\lambda l^2}{4h}$, and the total deflection will equal

$$\frac{3\lambda l^2}{4h} + \frac{\lambda l^2}{4h} = \frac{\lambda l^2}{h} \quad \ldots\ldots [1].$$

The deflection at the centre of a girder of span $2l$, and uniform cross-section, under a distributed load of w per unit length

$$= \frac{5wl^4}{24EI} \quad \ldots\ldots [2],$$

$$\text{and } M_{ff} = \frac{wl^2}{2} = \frac{rI}{y} = \frac{2rI}{d},$$

where d is the depth of the girder.

$$\therefore \text{ the deflection} = \frac{5rl^2}{6dE} \quad \ldots\ldots [3].$$

TEMPERATURE STRESSES.

If the temperature falls, the shortening of the cables will be (say) Δ per unit of length, and the rise of the cables at the centre will

$$= \frac{\Delta l^2}{h} \quad \ldots\ldots [4].$$

This rise is accompanied by an upward deflection of the girders, which the dead load tends to resist, and this resistance is equivalent to an uniform load of (say) K per unit length of span (*Fig. 2*).

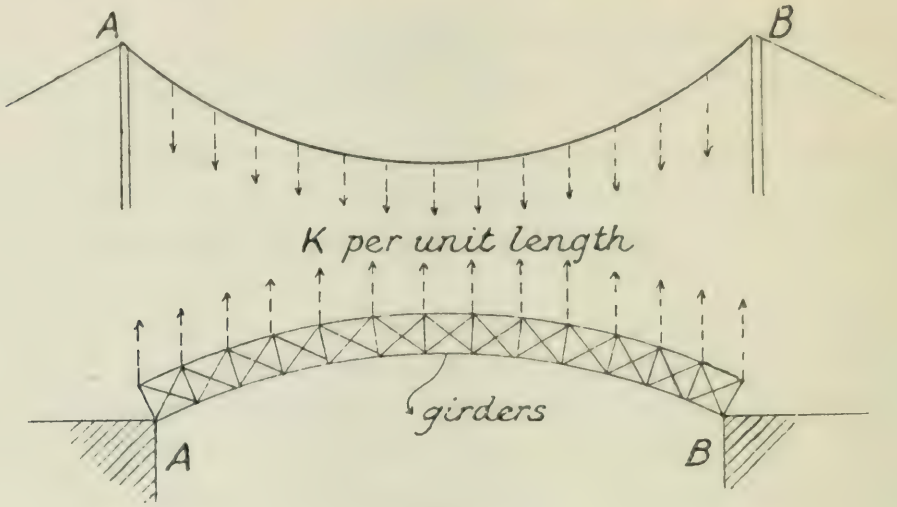

FIG. 2.

The horizontal tension in the cables due to this load is given by the equation

$$H = \frac{K l^2}{2h} \quad \ldots\ldots [5].$$

Let C be the cross-section of the cables, then the elongation per unit length, ∂, due to this load

$$= \frac{H}{CE} = \frac{K l^2}{2hCE} \quad \ldots\ldots [6],$$

and therefore the *downward* deflection of the cables at the centre

$$= \frac{\partial l^2}{h} = \frac{K l^4}{2h^2 CE} \quad \ldots\ldots [7].$$

The net *upward* deflection therefore

$$= (\Delta - \partial)\frac{l^2}{h} \quad \ldots\ldots [8],$$

$$= \frac{\Delta l^2}{h} - \frac{K l^4}{2h^2 CE} \quad \ldots\ldots [9],$$

and this obviously must equal the upward deflection of the girders, which is

$$= \frac{5 K l^4}{24 EI} \quad \ldots\ldots [10].$$

Equating [9] and [10] we obtain

$$\frac{\Delta l^2}{h} - \frac{K l^4}{2h^2 CE} = \frac{5 K l^4}{24 EI}, \text{ or } K = \frac{\frac{\Delta}{h}}{\frac{l^2}{2h^2 CE} + \frac{5 l^2}{24 EI}} \quad \ldots\ldots [11].$$

If G be the cross-sectional area of the flanges of the girders (a constant quantity), $I=\frac{Gd^2}{4}$, where d is the depth of the girders.

$$\therefore \quad K=\frac{2\Delta EhC}{l^2\left(1+\frac{5h^2C}{3d^2G}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots [12].$$

N.B.—It will be noted that the same value for E is assumed for both cables, and girders, and also that the elongation or contraction of the vertical suspension rods has been neglected.

Now the co-efficient of expansion of steel is $\frac{1}{150000}$ for one degree Fahrenheit, and the range of temperature to which the bridge will be exposed may be taken at 100° F., equivalent to a mean variation of 50° F.

The lengths of the vertical suspension rods would be adjusted so that at the mean temperature, and under the permanent load, the girders would be horizontal and consequently exposed to no stresses under these conditions. With a mean variation of 50° F., $\Delta=\frac{50}{150000}=\frac{1}{3000}$.

The span is 200 ft., the dip is $\frac{1}{12}$ span, and we may assume that the girder would be about 4 ft. deep, with a total cross-sectional area of flanges equal to about 16 square inches, and double the cross-sectional area of the cables.

Hence $l=1200''$, $h=200''$, $d=50''$, C = 8 square inches, G = 16 square inches, and E = 30,000,000 lbs.

From equation [12]

$$K=\frac{2\Delta EhC}{l^2\left(1+\frac{5h^2C}{3d^2G}\right)}=1{\cdot}55 \text{ lbs. per inch-run.}$$

The maximum temperature load therefore = 18·6 lbs. per foot-run, and $K=\pm 1{\cdot}55$ ..... [13].

A rise of temperature of 50° F. above the mean loads the girders with nearly 20 lbs. per foot-run, and unloads the cables to the same extent; while a fall of 50° F. below the mean has exactly the opposite effect.

From equations [3] and [10] the maximum temperature deflection of the girders is

$$\pm\frac{5Kl^4}{24EI}=\frac{5rl^2}{6dE},$$

whence $r=1{\cdot}25$ tons per square inch..... [14], while in the cables the maximum temperature horizontal tension $=\frac{Kl^2}{2h}$ (from equation [5]) = 5,580 lbs., with a corresponding intensity of $\frac{5580}{8}=697$ lbs. $=0{\cdot}31$ tons per square inch ..... [15].

PERMANENT LOAD STRESSES.

As stated above, the vertical suspension rods would be so adjusted that the girders would be horizontal, and free from stress at the mean temperature and under the permanent load.

The horizontal tension in the cables would be $\frac{pl^2}{h}$, where p is the permanent load per unit length.

Should the conditions, assumed above, be not absolutely attained in reality, then the girders would carry a portion of the permanent load by their own stiffness, and the cables would carry the remainder.

Let np be that portion carried by the cables, and $p(1-n)$ be the portion carried by the girders.

From equation [7] the deflection at the centre of the cables

$$= \frac{npl^4}{2h^2CE} \quad \ldots\ldots [16],$$

and from equation [10] the deflection at the centre of the girders

$$= \frac{5p(1-n)l^4}{6a^2GE} \quad \ldots\ldots [17],$$

since $I = \frac{Ga^2}{4}$.

Equating [16] and [17] we obtain

$$n = \frac{1}{1 + \frac{3a^2G}{5h^2C}} \quad \ldots\ldots [18],$$

whence $n = 0{\cdot}93 \ldots\ldots [19]$.

With a dead load of 20 lbs. per inch-run, the cables would carry 18·6 lbs., and the girders 1·4 lbs. per inch-run; with corresponding stresses of 3·74 tons per square inch in the cables (from equation [5]).....[20]
and 1·12 tons per square inch in the girder flanges .....[21]
from equations [3] and [10].

MOVING LOAD STRESSES.

Let the moving load be w per unit length of span. The stiffening girders prevent the cables from being distorted appreciably under the influence of an unsymmetrical load; and therefore as the cables always retain their parabolic form without material distortion, the effect of the stiffening girders is to distribute uniformly along the cables that portion of the moving load that is carried by the cables.

In *Fig.* 3 an increment, wdx, of the moving load is situated at a distance x from the centre.

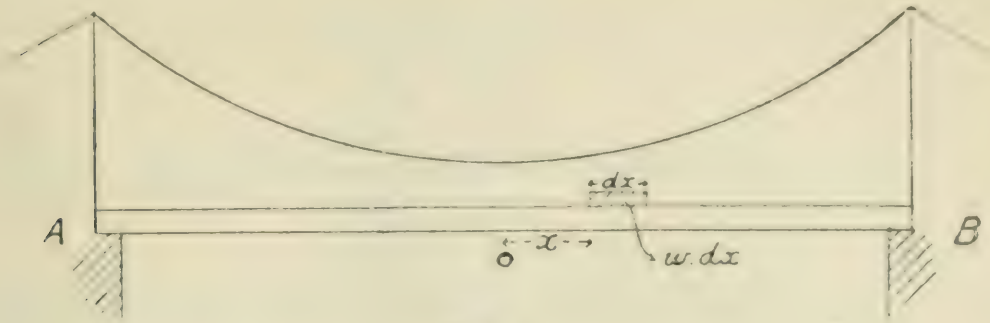

FIG. 3.

Let the proportion borne by the cables be wdn per unit length, equally distributed over the span.

From equation [7] the deflection of the cables at their centre, due to this load

$$=\frac{wdnl^4}{2h^2CE} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [22].$$

The deflection of the girders at their centre will be that due to the concentrated load, wdx, situated at a distance, x, from the centre, minus the *upward* deflection due to the equally distributed load, or upward tension in the suspension rods, of wdn per unit length.

Now the deflection at the centre of the girders due to a concentrated load of wdx, at a point distant x from the centre,

$$=\frac{wdx\,(2l^3-3lx^2+x^3)}{12\,EI};$$

while from equation [10] above, the deflection due to the equally distributed upward force of wdn per unit length

$$=\frac{5wdnl^4}{24\,EI}.$$

The downward deflection of the girders therefore

$$=\frac{wdx\,(2l^3-3lx^2+x^3)}{12\,EI}-\frac{5wdnl^4}{24\,EI} \quad \ldots\ldots\ldots\ldots [23].$$

Equating [22] and [23] we get

$$\frac{dx\,(2l^3-3lx^2+x^3)}{6\,I}-\frac{5dnl^4}{12\,I}=\frac{dnl^4}{h^2C},$$

and substituting $\frac{Gd^2}{4}$ for I we get

$$dn=\frac{(2l^3-3lx^2+x^3)\,dx}{l^4\left(\frac{5}{2}+\frac{3Gd^2}{2h^2C}\right)} \quad \ldots\ldots\ldots\ldots [24].$$

For the moving load distributed over a portion of the span, x_2-x_1, we get

$$\int_{n_1}^{n_2} dn = \frac{\int_{x_1}^{x_2} (2l^3 - 3lx^2 + x^3)\, dx}{l^4\left(\frac{5}{2} + \frac{3Gd^2}{2h^2C}\right)} \quad \ldots\ldots\ldots\ldots [25],$$

or
$$(n_2 - n_1) = \frac{\{2l^3(x_2-x_1) - l(x_2^3 - x_1^3) + \frac{1}{4}(x_2^4 - x_1^4)\}}{l^4\left(\frac{5}{2} + \frac{3Gd^2}{2h^2C}\right)} \ldots\ldots [26],$$

i.e.,
$$(n_2 - n_1) = \frac{\{2l^3(x_2-x_1) - l(x_2^3 - x_1^3) + \frac{1}{4}(x_2^4 - x_1^4)\}}{l^4\left(\frac{43}{16}\right)} \ldots\ldots [27].$$

If half the span be covered by the moving load so that

$$x_1 = 0, \text{ and } x_2 = l,$$

$$(n_2 - n_1) = \frac{20}{43} = 0{\cdot}465,$$

while if the whole span be covered, the value of (n_2-n_1) will be $\frac{2 \times 20}{43} = 0{\cdot}93$, the same value as in [19].

Figs. 4 and 5 represent the forces acting on the girders when the moving load covers the portion x_2-x_1 of the span.

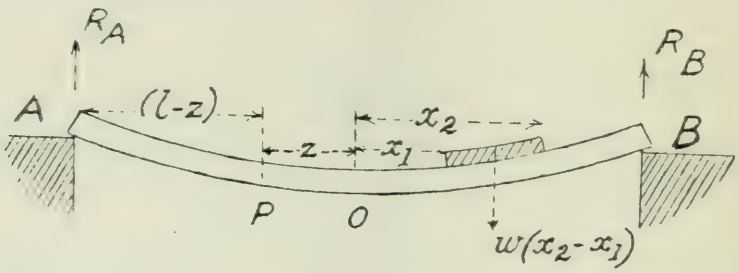

FIG. 4.

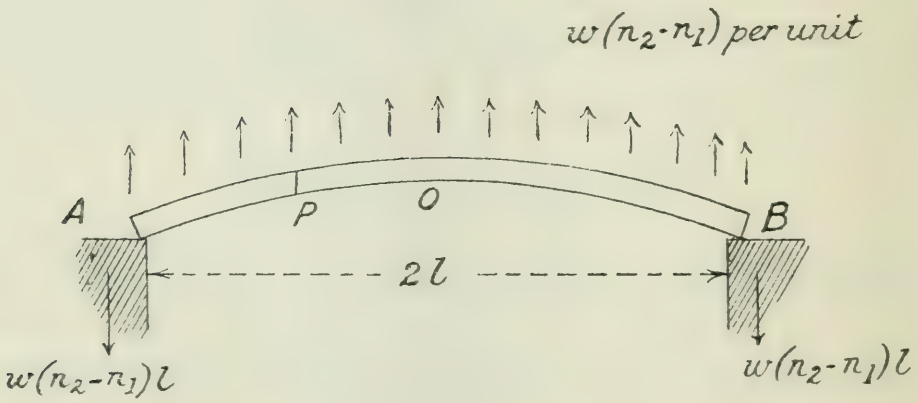

FIG. 5.

From equation [27] we can get values for $(n_2 - n_1)$ for either half of the span as follows :—

TABLE I.

$x_1 = 0$		$x_1 = l$	
$\frac{x_2}{l}$	$(n_2 - n_1)$	$\frac{x_1}{l}$	$(n_2 - n_1)$
0·1	·074	0	·465
0·2	·146	0·1	·391
0·3	·214	0·2	·319
0·4	·276	0·3	·251
0·5	·331	0·4	·189
0·6	·378	0·5	·134
0·7	·415	0·6	·087
0·8	·443	0·7	·049
0·9	·460	0·8	·022
1·0	·465	0·9	·006

Ex.—If the moving load covers the span as in *Fig.* 6

$$(n_2 - n_1) = 0{\cdot}415 + 0{\cdot}276 = 0{\cdot}691.$$

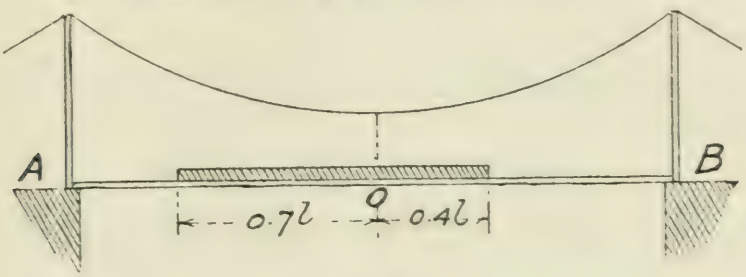

FIG. 6.

If the load covers the span as in *Fig.* 7

$$(n_2 - n_1) = (0{\cdot}331 - 0{\cdot}146) + 0{\cdot}189 = 0{\cdot}374,$$

and similarly for any possible distribution of the moving load.

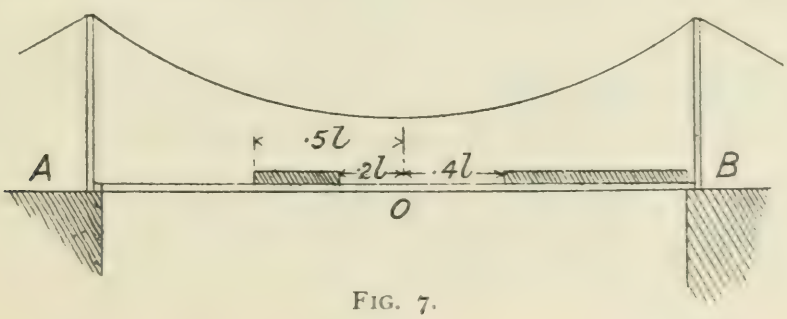

FIG. 7.

Let a concentrated load, W, be placed on the girders at a point distant x from the centre; and let the corresponding distributed load on the cables be q per unit length of span.

Fig. 8 shows the various forces and reactions acting on the girders.

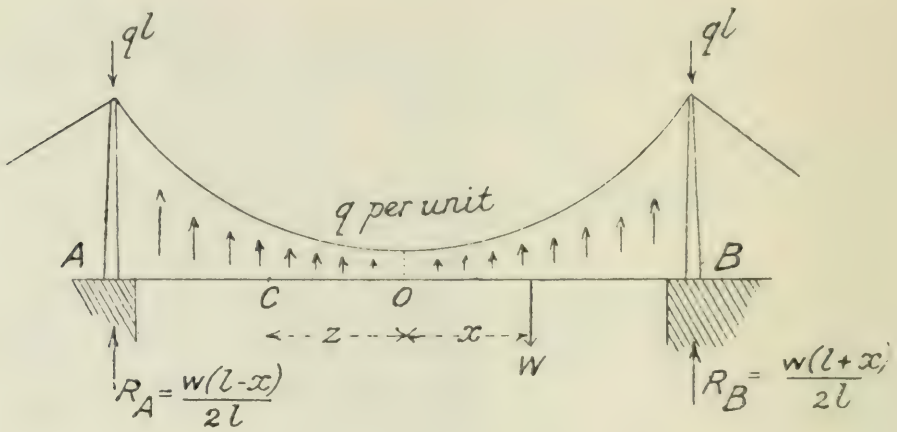

FIG. 8.

The deflection downwards of the girders at the centre

$$= \frac{W\,(2l^3 - 3lx^2 + x^3)}{12EI} - \frac{5ql^4}{24EI} \quad \ldots\ldots\ldots\ldots [30].$$

The deflection of the cables at the centre

$$= \frac{ql^4}{2h^2CE} \quad \ldots\ldots\ldots\ldots [31].$$

Substituting numerical values for I, h, and C, and equating [30] and [31], we obtain

$$43ql^4 = 16W\,(2l^3 - 3lx^2 + x^3) \quad \ldots\ldots\ldots\ldots [32].$$

At any point C, on the left half of the span, distant z from O,

$$M_f = -\frac{W\,(l-x)\,(l-z)}{2l} + ql\,(l-z) - q\,\frac{(l-z)^2}{2} \quad \ldots\ldots [33].$$

Substituting for q the value from [32] and putting $M_f = 0$, we get

$$(l+z) = \frac{43\,(l-x)\,l^3}{16\,(2l^3 - 3lx^2 + x^3)} \quad \ldots\ldots\ldots\ldots [34].$$

This equation gives the points where the M_f=zero for any given position of one load. If W is on the same side of the centre as the point C, then $R_A = W\,\frac{(l+x)}{2l}$, and we must write $(l+x)$ for $(l-x)$ in the numerator of equation [34].

From this equation we get the following table with reference to *Fig.* 8, measurements of x from O towards B, and of z from O towards A being considered positive, and *vice versâ*.

TABLE II.

$\frac{x}{l}$	$\frac{z}{l}$
0·9	−0·10
0·8	−0·09
0·7	−0·08
0·6	−0·05
0·5	−0·02
0·4	0·02
0·3	0·07
0·2	0·14
0·1	0·23
0	0·34
−0·1	0·50
−0·2	0·71
−0·3	0·99

Thus (for example) all loads to the right of the point $x=0{\cdot}6l$ cause hogging bending moments between the left abutment and a point situated $0{\cdot}05l$ to the *right* of the centre.

Similarly the maximum sagging M_f at the centre is caused by loads extending to a distance of $0{\cdot}45l$ on each side of the centre; and the maximum hogging M_f at the centre is caused by loads extending from the abutments inwards for a distance of $0{\cdot}55l$ on either half-span.

We can now calculate the maximum M_f at the centre of the girders; and a hogging M_f will be considered positive, and *vice versâ*.

(1). The maximum hogging M_f at the centre.

In this case the load covers the span from either abutment for a space of $0{\cdot}55l$ inwards.

From equation [27] for either half of this loading

$$(n_2 - n_1) = \frac{\frac{1}{2}l^3\ (l - 0{\cdot}45l) - l\ (l^3 - 0{\cdot}45^3 l^3) + \frac{1}{4}\ (l^4 - 0{\cdot}45^4 l^4)\frac{1}{4}}{\frac{43}{16}\ l^4}$$

$$= 0{\cdot}16 \ldots [35].$$

$$\therefore \qquad 2\ (n_2 - n_1) = 0{\cdot}32.$$

(N.B.—This could be obtained from Table I. by interpolation).

At the centre (*Fig.* 9)

$$M_f = -\ 0{\cdot}55wl^2 + 0{\cdot}32wl^2 + (0{\cdot}55 \times 0{\cdot}725)\ wl^2 - 0{\cdot}32\ \frac{wl^2}{2}$$

$$= +\cdot 009wl^2 \ldots\ldots [36].$$

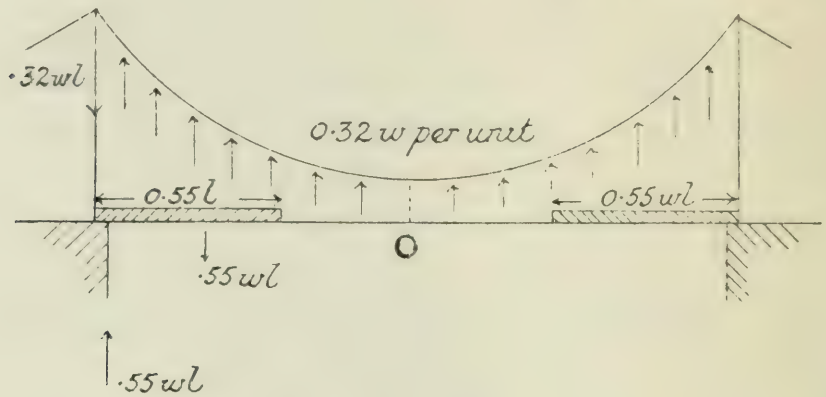

FIG. 9.

(2). In a similar manner we obtain for the maximum sagging M_f at the centre,

$$2\ (n_2 - n_1) = 0{\cdot}61 \text{ (from Table I. by interpolation),}$$

and

$$M_f = -\ 0{\cdot}45wl^2 + 0{\cdot}61wl^2 + \left(0{\cdot}45 \times \frac{0{\cdot}45}{2}\right)\ wl^2 - 0{\cdot}61\ \frac{wl^2}{2}$$

$$= -\cdot 044wl^2 \ldots\ldots [37].$$

(3). The maximum hogging bending moment at *any* point in the girders is at the point $z = 0{\cdot}6l$ (*vide Fig.* 8), when the load covers the span from the other abutment to a point $0{\cdot}15l$ beyond the centre (*vide* Table II.) as shown in *Fig.* 10.

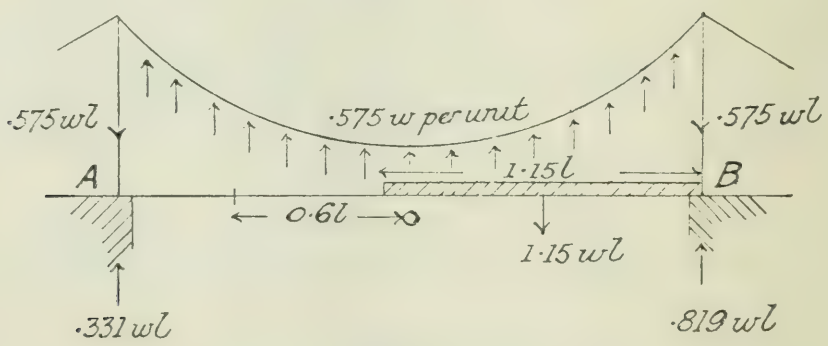

FIG. 10.

$$(n_2 - n_1) = 0{\cdot}465 + 0{\cdot}110 = 0{\cdot}575 \text{ (from Table I.).}$$

$\therefore$ M_f at this point, $(z=0{\cdot}6l)$,

$$=-(0{\cdot}331\times0{\cdot}4)\,wl^2+(0{\cdot}575\times0{\cdot}4)\,wl^2-\frac{{\cdot}575\times(0{\cdot}4)^2}{2}\,wl^2$$

$$=+{\cdot}0516wl^2\ldots\ldots[38].$$

(4). In the same way the maximum sagging bending moment at *any* point in the girders is at the point $z=0{\cdot}5l$, and when the load is as in *Fig.* 11 (*vide* Table II.).

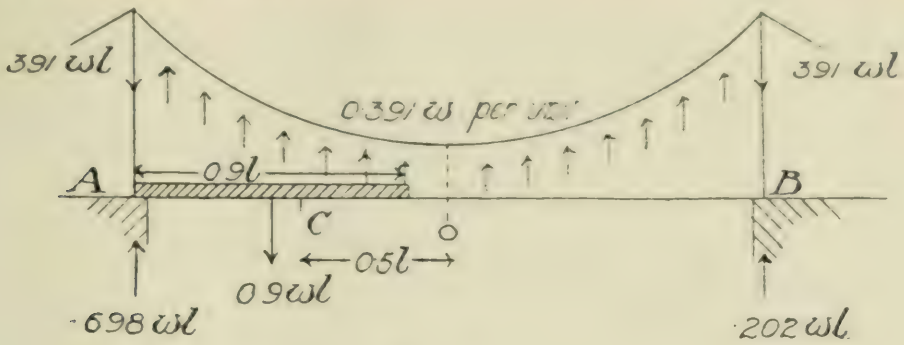

FIG. 11.

From Table I., $(n_2-n_1)=0{\cdot}391$.

$\therefore$ M_f at C

$$=\left\{(0{\cdot}391-0{\cdot}698)\times0{\cdot}5wl^2\right\}+\left\{(1-0{\cdot}391)\times\frac{(0{\cdot}5)^2\,wl^2}{2}\right\}$$

$$=-{\cdot}0774wl^2\ldots\ldots[39].$$

We find, therefore, that the greatest sagging bending moment in the girders due to the moving load is $-{\cdot}0774wl^2$, and the greatest hogging bending moment is $+{\cdot}0516wl^2$.

With a moving load equivalent to 25 lbs. per inch-run

$$M_{ff}={\cdot}0774wl^2={\cdot}0774\times25\times1200^2=1{,}244 \text{ inch-tons}\ldots[40].$$

$$M_r=\frac{rI}{y}=\frac{2rI}{d}=\frac{2rGd^2}{4d}=\frac{rGd}{2}=M_{ff}.$$

$$G=16 \text{ square inches}, \quad d=50 \text{ inches}.$$

$\therefore$ $$M_r=\frac{r\times16\times50}{2}=1{,}244 \text{ inch-tons}.$$

$\therefore$ $$r=3{\cdot}11 \text{ tons per square inch} \ldots\ldots\ldots\ldots[41],$$

where r=maximum intensity of stress in the girder flanges.

The maximum stress in the cables is, of course, caused when the moving load covers the whole span; in this case $n=0{\cdot}93$ [equation 19], and the horizontal tension in the cables [equation 5] $=\frac{0{\cdot}93wl^2}{2h}=37{\cdot}37$ tons,

or $$\frac{37{\cdot}37}{8}=4{\cdot}67 \text{ tons per square inch} \ldots\ldots\ldots[42].$$

WIND STRESSES.

To counteract wind pressure, a horizontal cable would be stretched from bank to bank on either side of the bridge, connected by rods to the *bottom* booms of the stiffening girders. *Fig.* 12 shows the arrangement in *plan*.

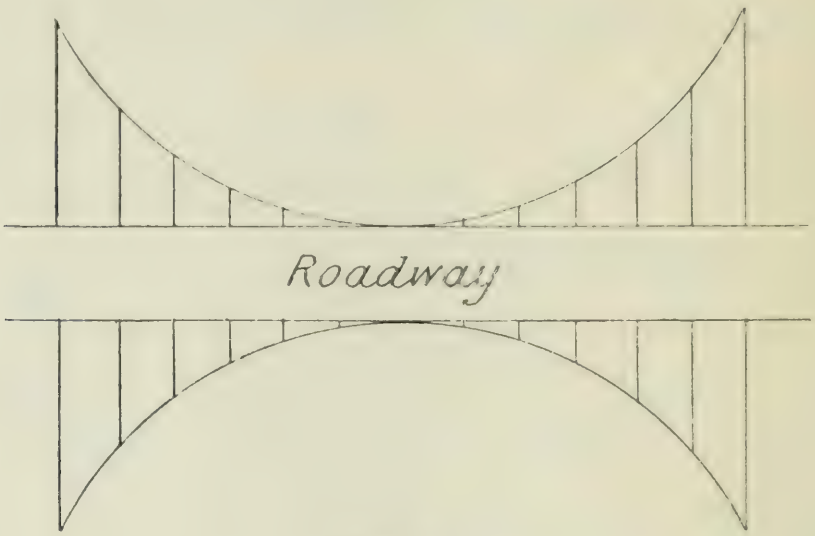

FIG. 12.

The wind pressure may be considered as a uniform distributed load and the *two bottom booms of the stiffening girders with the horizontal bracing beneath the roadway* will act as a stiffening girder to the wind cables.

The wind pressure may be taken at 60 lbs. per foot-run of the span, or 5 lbs. per inch-run; the cross-sectional area of either wind cable may be taken as 1 square inch; the cross-sectional area of the flanges of the stiffening girder will be $G_1 = \frac{G}{2} = 8$ square inches; $l = 1{,}200''$ as before, and h may be taken as $300''$, since there is no objection to having a high value of h compared with the span in this case. The depth of the girder will be the breadth of the roadway, or $180''$.

In the case of the wind cables there are no back guys, and consequently the deflection at their centre (*vide* [A], page 9) is

$$\frac{3\lambda l^2}{4h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [43].$$

Calling the wind load w per unit length, the wind cable will take nw, and the girder $(1-n)w$ per unit length.

The horizontal tension in the cable, H, due to this load is (*vide* [5]),

$$H = \frac{nwl^2}{2h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [44],$$

and the elongation per unit length, $\lambda = \frac{H}{CE}$ (*vide* [6]).

Therefore from [43] the deflection of the cable at the centre

$$= \frac{3\Delta l^2}{4h} = \frac{3Hl^2}{4hCE} = \frac{3nwl^4}{8h^2CE} \quad \ldots\ldots\ldots\ldots\ldots\ldots [45].$$

The deflection of the girder at the centre, from [10],

$$= \frac{5(1-n)wl^4}{24EI} \quad \ldots\ldots\ldots\ldots\ldots\ldots [46]$$

$$\left\{ l = 1200'', h = 300'', C = 1 \text{ square inch}, I = \frac{G_1 d_1^2}{4} = \frac{8 \times 180^2}{4} = 2 \times 180^2 \right\}.$$

Equating [45] and [46] we obtain

$$\frac{3n}{8h^2C} = \frac{5(1-n)}{24I}, \text{ whence } n = 0{\cdot}44 \ldots\ldots\ldots\ldots [47].$$

From [3] and [10] the maximum deflection of the girder, due to this load,

$$= \frac{5(1-n)wl^4}{24EI} = \frac{5rl^2}{6dE},$$

whence $r = 1000(1-n)w = 1000 \times 0{\cdot}56 \times 5 = 2{,}800$ lbs. per square inch; *i.e.*, 1·25 tons per square inch is the maximum flange stress of the girder due to wind pressure ..... [48]
while the horizontal tension in the cable due to the same cause is, from [44],

$$\frac{nwl^2}{2h} = \frac{0{\cdot}44 \times 5 \times l^2}{2h} = 5{,}280 \text{ lbs.},$$

or 2·36 tons per square inch ..... [49].

Now to ensure the wind cables doing their work properly, they must never be slack; and consequently they must be just taut at the highest temperature. When the temperature falls, the wind cables pull against each other and prevent any contraction, setting up tension in the horizontal rods connecting them to the horizontal girder. Consequently there is a permanent temperature stress in the wind cables, except when the temperature is at its maximum. From page 11 we see that the maximum range is 100° F., and therefore

$$\Delta = \frac{100}{150{,}000} = \frac{1}{1{,}500}.$$

The maximum "upward" deflection, due to this cause, if the cable were free, (*vide* [43]), would be

$$\frac{3\Delta l^2}{4h} \quad \ldots\ldots\ldots\ldots\ldots\ldots [50].$$

This deflection "upwards" is prevented by the horizontal rods, equivalent to a uniform load, say m, per unit length.

The deflection "downwards," due to this load, *vide* equation [45], is

$$\frac{3ml^4}{8h^2CE} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [51].$$

Equating [50] and [51] we get

$$\Delta = \frac{ml^2}{2hCE} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [52].$$

$\Delta = \frac{1}{1,500}$, $h = 300''$, C = 1 square inch, E = 30,000,000 lbs.

$$\therefore \qquad m = \frac{25}{3} \text{ lbs. per inch-run} \quad \ldots\ldots\ldots\ldots\ldots\ldots [53],$$

and the consequent horizontal tension due to this load (from [44])

$$= \frac{ml^2}{2h} = 20{,}000 \text{ lbs., or } 8{\cdot}93 \text{ tons per square inch} \quad \ldots\ldots [54].$$

MAXIMUM TENSION IN WIND CABLES.

The total maximum horizontal tension in the wind cables is given by [49] and [54], viz., $2{\cdot}36 + 8{\cdot}93 = 11{\cdot}29$ tons per square inch; and the maximum tension at the abutments

$$= 11{\cdot}29 \sec\phi = 11{\cdot}29 \times 1{\cdot}118 = 12{\cdot}62 \text{ tons per square inch} \ldots [55].$$

(N.B.—Tan $\phi = \frac{2h}{l} = \frac{1}{2}$, where ϕ is the angle made by the tangent to the parabola with the "horizontal" at the abutments).

This is not excessive, as the material would be plough steel, and the maximum stress would rarely occur; for it is due to a combination of the maximum wind pressure and the coldest temperature.

MAXIMUM TENSION IN MAIN CABLES.

The maximum horizontal tension in the main cables is composed of that due to (*a*) temperature, (*b*) permanent load, and (*c*) moving load.

From [15] we see that (*a*) is 0·31 tons per square inch.
" [20] " " (*b*) is 3·74 " " "
" [42] " " (*c*) is 4·67 " " "

making a total of 8·72 tons per square inch.

At the standards the maximum tension in the cables is $8{\cdot}72 \sec\phi$, or

$$8{\cdot}72 \times 1{\cdot}054 = 9{\cdot}19 \text{ tons per square inch} \quad \ldots\ldots\ldots\ldots [56]$$

$$\left(\tan\phi = \frac{2h}{l} = \frac{1}{3}\right).$$

This is not too great a working stress for good quality plough steel rope.

The cross-section of the back guys would be proportional to the stress that they sustain.

The length of one back guy has been taken at $\frac{l}{3}$ (*vide* page 9), and assuming the height of the standards to be $h + 50 = 250''$, the slope

of the guys is very nearly $\frac{3}{5}$, and the secant of the angle they make with the horizontal about $\frac{4}{3}$. Their cross-sectional area therefore must be one-third more than that of the cables.

The small difference in the tensions of the main cables at their centre and at the standards is negligible. The standards might be pivoted at their bases to avoid any movement of the cables over their heads due to alterations of length in the back guys caused by changes of temperature.

MAXIMUM STRESSES IN GIRDER FLANGES.

The maximum stresses in the girder flanges, due to temperature, permanent load, and wind pressure, are at the centre ; for all these loads are uniformly distributed ; and the bending moments caused by them (and consequently the stresses) vary as the ordinates of a parabola.

The maximum stress, caused by the moving load is at points situated $0{\cdot}5l$ on either side of the centre, *vide* [39], and the true point of maximum stress due to a combination of all possible loads must be situated therefore rather nearer the centre.

The temperature load, *vide* [14], causes a stress of 1·25 tons per square inch in the girder flanges, the permanent load, *vide* [21], causes a stress of 1·12 tons per square inch, and wind pressure, *vide* [48], causes a stress of 1·25 tons per square inch.

Now the wind pressure acts on a girder formed by the two bottom flanges of the stiffening girders and the horizontal roadway bracing. The stress it causes in the bottom flanges is exactly the same as the temperature stress, viz., 1·25 tons per square inch. Consequently the same load on the upper flanges, (viz., 1·55 lbs. per inch, *vide* [12]), would cause the wind stress on the bottom flanges. The same result would be arrived at as follows :—

Depth of stiffening girder, 50″.
Flange area „ „ 16 square inches.
Depth of wind girder, 180″.
Flange area, „ „ 8 square inches.
Wind load on „ „ 5 (1 − 0·44) lbs. per inch-run = 2·80 lbs., *vide* [47].

Therefore an equivalent load on the top flanges of the stiffening girders would be

$$\frac{2{\cdot}80 \text{ lbs.} \times 50'' \times 16 \text{ sq. ins.}}{180'' \times 8 \text{ sq. ins.}} = 1{\cdot}55 \text{ lbs. per inch-run.}$$

The equivalent load on the stiffening girders therefore is

(*a*). Temperature load ... 1·55 lbs.
(*b*). Permanent load ... 1·40 lbs. (*vide* page 12).
(*c*). Wind load ... ... 1·55 lbs.

making a total of 4·50 lbs. per inch-run.

Referring to *Fig.* 8, the M_f at C due to these loads is

$$-4{\cdot}50l(l-z)+\frac{4{\cdot}50(l-z)^2}{2}=-\frac{4{\cdot}5(l^2-z^2)}{2}\ldots\ldots\ldots\ldots[57].$$

When $z=0{\cdot}5l$, the M_f due to these loads $=-1{\cdot}69l^2$, while the M_f due to the moving load, *vide* [39] $=-{\cdot}0774wl^2$, or $-1{\cdot}935l^2$, since $w=25$ lbs.

$\therefore$ M_{ff} at this point, $z=0{\cdot}5l$, is $(-1{\cdot}69l^2-1{\cdot}935l^2)=-3{\cdot}625l^2$.

When $z=0{\cdot}4l$, the M_f due to temperature, permanent and wind loads $=-1{\cdot}89l^2$, while the M_f due to the moving load $=-1{\cdot}882l^2$, for in this case we obtain from Tables I. and II., $(n_2-n_1)=0{\cdot}435$, and the moving load covers the span from A for a length of $0{\cdot}96l$.

$\therefore$ M_{ff} at this point, $z=0{\cdot}4l$, is $-3{\cdot}772l^2\ldots\ldots\ldots\ldots[58]$.

When $z=0{\cdot}3l$, $(n_2-n_1)=0{\cdot}495$, the load extends from A for a distance of $1{\cdot}04l$, and $M_{ff}=(-2{\cdot}05l^2-1{\cdot}72l^2)=-3{\cdot}770l^2$. Consequently M_{fff} is at points situated $0{\cdot}4l$ on either side of the centre, and

$$=-3{\cdot}772l^2=-2{,}425 \text{ inch-tons} \ldots\ldots\ldots\ldots[59].$$

$$M_{fff}=2{,}425=M_r=\frac{rI}{y}=\frac{rGd}{2}=\frac{r\times 16\times 50}{2},$$

whence $r=6{\cdot}06$ tons per square inch $\ldots\ldots\ldots\ldots$ [60],

which is within the working limit of 6·5 tons per square inch in compression for mild steel, and will permit of sufficient rivet holes in the flanges, since the working tensile stress is 7·5 tons per square inch.

The area of each flange of the stiffening girder is of course $\frac{16}{4}=4$ square inches.

As the upper flanges of the stiffening girders are not exposed to any wind load, the maximum stress in their case will be about 1 ton per square inch less.

SHEARING STRESSES.

The maximum shearing stresses in the stiffening girders will be at the abutments.

That due to the temperature load, *vide* [13], will be

$$\pm 1{\cdot}55l \text{ lbs.} \ldots\ldots\ldots\ldots[61].$$

That due to the permanent load, *vide* [19], will be

$$+1{\cdot}4l \text{ lbs.} \ldots\ldots\ldots\ldots[62].$$

That due to the moving load can be investigated as follows :—

The shearing force at the left abutment A (*Fig.* 8), due to a single load, W, placed on the right half of the span at a distance x from the centre, will be

$$+\frac{W(l-x)}{2l}-ql \ldots\ldots\ldots\ldots[63].$$

At any point C, on the left half of the span, distant z from the centre, the shearing force, S,

$$= + \frac{W(l-x)}{2l} - ql + q(l-z).$$

If C be the point where the shearing force is zero, we have

$$S = \frac{W(l-x)}{2l} - qz = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots [64].$$

Now from [32] we know that

$$q = \frac{16\,W(2l^3 - 3lx^2 + x^3)}{43l^4}.$$

Substituting this value of q in [64] we obtain

$$z = \frac{43l^3(l-x)}{32(2l^3 - 3lx^2 + x^3)} \quad \ldots\ldots\ldots\ldots\ldots\ldots [65].$$

This equation gives the points where the shearing force is zero, for any given position of one load. If W is on the left half span, the same side as C, then $(l+x)$ must be written for $(l-x)$ in the numerator of equation [65].

From this equation we get the following table :—

Table III.

$\frac{x}{l}$	$\frac{z}{l}$
·9	·45
·8	·45
·7	·46
·6	·47
·5	·49
·4	·51
·3	·54
·2	·57
·1	·61
·0	·67
−·1	·75
−·2	·85
−·3	1·00

From Table III. we see that the arrangements of the loads which will give the maximum negative and positive shearing stresses at the left abutment are as shown in *Figs.* 13 and 14.

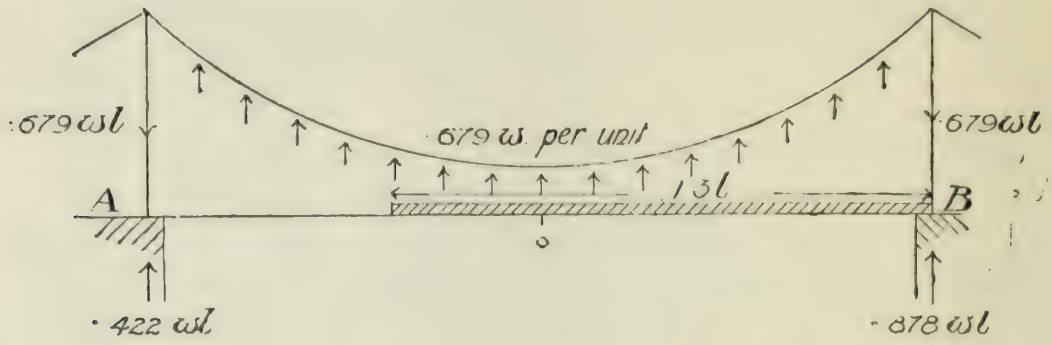

FIG. 13.

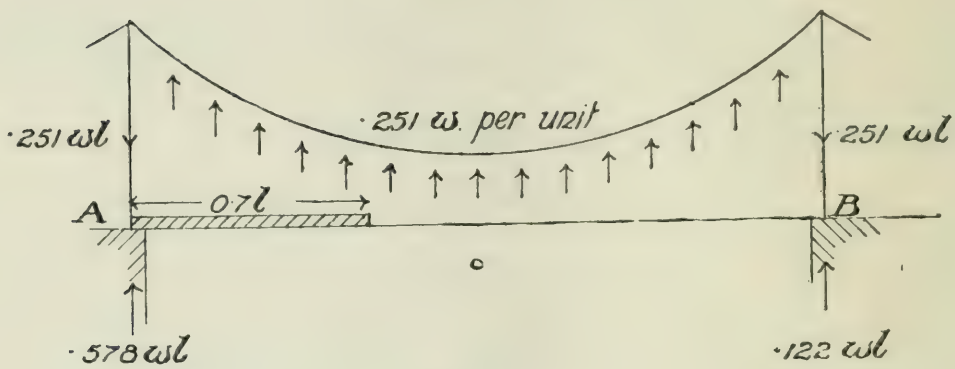

FIG. 14.

In the case of *Fig.* 13 we obtain from Table I. the value of $(\cdot 465+\cdot 214)=0\cdot 679$ for (n_2-n_1) ; and

$$S=+\cdot 422wl-\cdot 679wl=-0\cdot 257wl \quad \ldots\ldots\ldots\ldots [66].$$

For *Fig.* 14 we get a value of ·251 for (n_2-n_1) from Table I., and

$$S=+\cdot 578wl-\cdot 252wl=+0\cdot 327wl \quad \ldots\ldots\ldots\ldots [67].$$

$$w=25 \text{ lbs.}$$

$$\therefore \quad S=-6\cdot 42l \text{ or } +8\cdot 18l \ldots\ldots\ldots\ldots\ldots\ldots [66].$$

Combining this result with [61] and [62] we get the following maximum values for S, viz. :—

$$S=+1\cdot 55l+1\cdot 40l+8\cdot 18l=+11\cdot 13l \text{ lbs.}=+5\cdot 96 \text{ tons} \ldots [67].$$

$$S=-1\cdot 55l-6\cdot 42l=-7\cdot 97l \text{ lbs.}=-4\cdot 27 \text{ tons}\ldots\ldots\ldots\ldots\ldots [68].$$

It follows therefore that the ends of the girders must be secured by bolts to prevent their rising under certain conditions ; but the bolt holes must be slotted to allow of expansion and contraction. As the girders are in pairs the shearing stresses will be half these amounts.

If the panels of the girders are square, with *cross-braced* diagonals to resist tension only, then the maximum compression in the end vertical will be

$$\frac{5{\cdot}96}{2} = 2{\cdot}98 \text{ tons} \quad \ldots\ldots\ldots\ldots\ldots\ldots [69],$$

and the maximum tension in the end diagonals

$$2{\cdot}98\sqrt{2} = 4{\cdot}2 \text{ tons} \quad \ldots\ldots\ldots\ldots\ldots\ldots [70].$$

As the shearing stress at the abutments can be either plus or minus, the diagonals must be cross-braced.

In the wind girder the maximum shearing stress at the abutments (*vide* [47]) equals

$$\pm \tfrac{5}{2}(1-n)\,l \text{ lbs.} = \pm \frac{5 \times {\cdot}56 \times 1200}{2240} = \pm 1{\cdot}5 \text{ tons} \ldots\ldots [71].$$

As before let the short braces of the wind girder take compressive stresses only, and the cross-braced diagonals only tension.

If the panels are square the maximum tension in the diagonals is

$$1{\cdot}5\sqrt{2} \text{ tons} = 2{\cdot}12 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots [72].$$

The maximum compression in the short braces is

$$1{\cdot}5 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots [73],$$

but they may have to take a temperature tension, *vide* equation [53] of $8\frac{1}{3}$ lbs. per inch-run, or $\frac{25}{3} \times \frac{180}{1} = 1{,}500$ lbs. on a 15-foot panel. This is less than the maximum compressive stress and may be neglected.

The various data for designing the bridge are now complete.

ARTIFICIAL ADJUSTMENT OF STRESSES.

As the maximum sagging bending moment in the stiffening girders, *vide* [59], is $-3{\cdot}772l^2$, it is considerably greater than the maximum hogging bending moment, which is $+1{\cdot}29l^2$ caused by the moving load, (*vide* [38]), and $+\frac{2 \times 1{\cdot}55l^2}{2} = +1{\cdot}55l^2$, due to the wind and temperature loads, *vide* [57]. As however these two maxima bending moments do not occur at the same point, we will assume the maximum hogging bending moment to be

$$+2{\cdot}70l^2 \text{ approximately} \ldots\ldots\ldots\ldots\ldots\ldots [74].$$

If we give the stiffening girders an *artificial* camber by adjusting the vertical suspension rods, we shall increase the maximum hogging M_f, and decrease the sagging M_f, but will increase slightly the maximum tension in the cables.

For example the maximum deflection of the cables at their centre

due to temperature, [9 and 10], $=2{\cdot}23''$

„ permanent load, $\frac{nbl^4}{2h^2CE}$, [16], $=2{\cdot}01''$

„ moving load, $\frac{nwl^4}{2h^2CE}$, [22], $=2{\cdot}51''$.

Now if we give a camber of 2.01″, *so that the girders lie horizontal under the permanent load*, we relieve them of all stresses due to that load, and we find that M_x is at the points, $z=0{\cdot}4l$, and

$$=-3{\cdot}184l^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [75]$$

and $r=5{\cdot}12$ tons per square inch (*vide* page 24) .....[76].

The maximum tension in the cables will be increased slightly, as they now carry all the permanent load, instead of 0·93 of it.

Referring to page 22, we now see that the maximum horizontal tension

due to temperature, [15], is 0·31 tons per square inch,
„ permanent load, [5], is 4·02 „ „
„ moving load, [42], is 4·67 „ „

making a total of 9 tons per square inch, and with a maximum tension of

$9\times 1{\cdot}054=9{\cdot}49$ tons per square inch at the standards.....[77].

The *total* maximum tension in the back guys (*vide* page 23) will be

$$9\times 8\times\tfrac{4}{3}=96 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [78].$$

The maximum positive shearing stress will be modified also, as the permanent load causes no stress.

Referring to page 26, [67], etc., we now get

$$S=+1{\cdot}55l+8{\cdot}18l=+9{\cdot}73l \text{ lbs.}=+5{\cdot}21 \text{ tons} \ldots\ldots [79].$$

$$S=-1{\cdot}55l-6{\cdot}42l=-7{\cdot}97l \text{ lbs.}=-4{\cdot}27 \text{ tons} \ldots\ldots [80].$$

A slight further adjustment of the suspension rods could be made to equalize exactly the maximum positive and negative stresses, but this would not be worth while, as in practice it is much simpler to get the girders truly horizontal than to adjust them in an artificial upward curve.

CHAPTER III.

SUSPENSION BRIDGE WITH HINGED GIRDERS.

IF the stiffening girders are hinged at the middle, and anchored to the abutments as before, it is obvious that (i.) no temperature stresses can be set up, and (ii.) that the cables take the whole load. The part played by the girders, if they are stiff enough, being to distribute equally on to the cables any unsymmetrical load. Owing to the absence of temperature stresses the girders can be made as deep as is required, and great rigidity ensured. It is clear that no evenly distributed load will produce any stress in the girders, and only moving loads need be discussed.

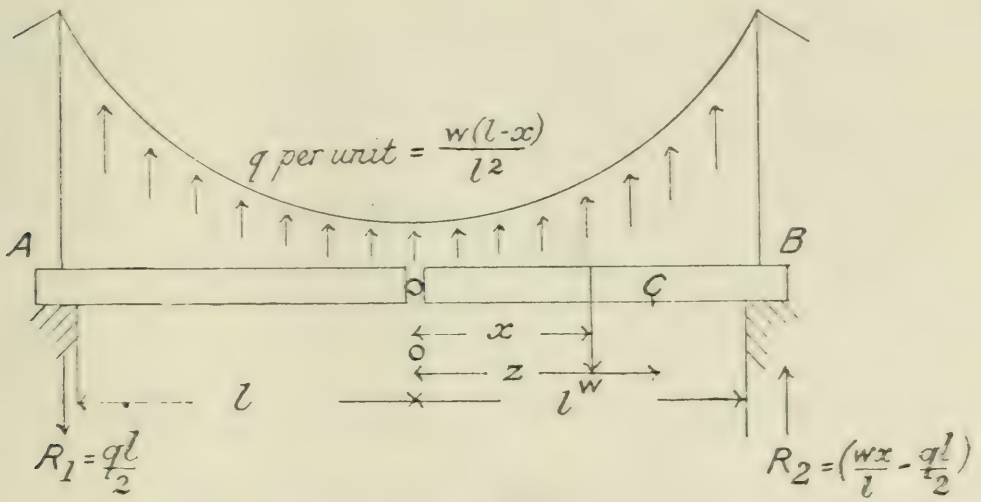

FIG. 15.

MOVING LOAD STRESSES.

As before, the tensions in the vertical rods are uniform, as the parabolic curve is not distorted appreciably. Also the M_f at the central hinge, *Fig.* 15, must always be zero. Let a concentrated load, W, be placed on the right half span. At O the $M_f = 0$; therefore (with reference to the *left* half span), taking moments round O, $R_1 \times l = \frac{ql^2}{2}$, whence R_1, (the reaction at A) $= \frac{ql}{2}$.

In a similar manner for the right half span, R_2, the reaction at B, $= \frac{Wx}{l} - \frac{ql}{2}$.

The sum of all the vertical forces acting on the structure must be zero, for equilibrium,

$$\therefore \quad -R_1 + 2ql - W + R_2 = 0,$$

whence $$q = \frac{W(l-x)}{l^2}.$$

$$\therefore \quad R_1 = \frac{W(l-x)}{2l}, \text{ and } R_2 = \frac{W(3x-l)}{2l}.$$

On the unloaded half span the maximum hogging bending moment occurs at its centre and equals $+\frac{ql^2}{8}$; and the bigger q is the bigger this M_f is; consequently a full load on the right half span causes the biggest hogging M_f on the left half span, at its centre, subject to the following remarks :—

On the right half span at any point C, distant z from the centre,

$$M_f = -R_2(l-z) - q\frac{(l-z)^2}{2}; \; z \text{ being } > x.$$

i.e. $$M_f = -\frac{Wx}{l}(l-z) + \frac{ql}{2}(l-z) - q\frac{(l-z)^2}{2} = -\frac{Wx}{l}(l-z) + \frac{qz}{2}(l-z).$$

At a point of contraflexure $M_f = 0$.

$$\therefore \quad -\frac{Wx}{l}(l-z) + \frac{qz}{2}(l-z) = 0,$$

and substituting for q, we have

$$-\frac{Wx}{l} + \frac{Wz(l-x)}{2l^2} = 0, \text{ whence } z = \frac{2lx}{l-x}.$$

Giving values to x, we obtain the following table :—

TABLE IV.

$\frac{x}{l}$	$\frac{z}{l}$
0	0
0·1	0·22
0·2	0·50
0·3	0·86
0·33	1·00

From this table we see that all loads, between the centre and the point $x = \frac{l}{3}$, cause a hogging bending moment at some point in that girder on which the loads are. This point of contraflexure shifts

very rapidly, and as the large maximum hogging M_f on the *unloaded* span is at its centre, we will add the additional amount due to a load extending from the centre to the point $x=0{\cdot}2l$.

For a full load on the right half span, *Fig.* 16, we have

$$R_1=\frac{ql}{2},\quad R_2=\frac{wl}{2}-\frac{ql}{2},\text{ and } q=\frac{w}{2}.$$

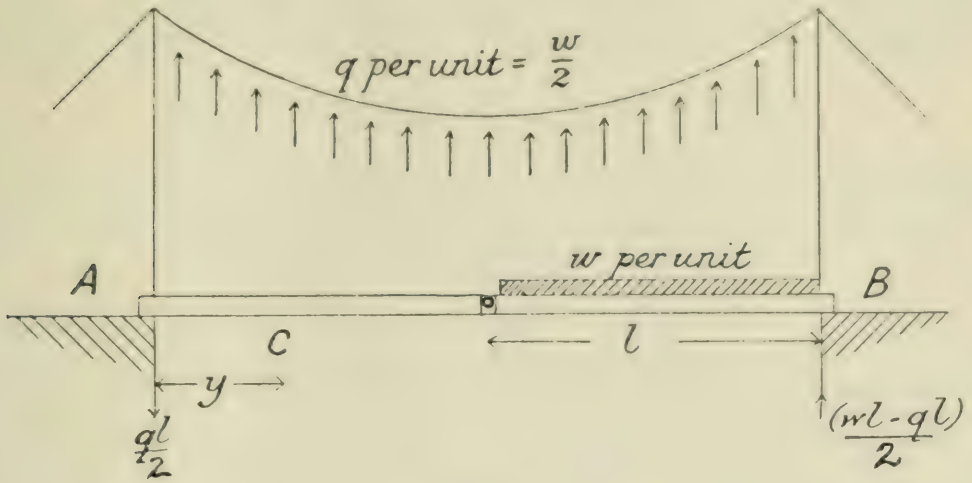

FIG. 16.

At any point C on the left half span, at a distance y from A,

$$M_f=\frac{qly}{2}-\frac{qy^2}{2}=\frac{wy}{4}(l-y).$$

For the load on the *left* half span, as in *Fig.* 17, we have

$$R_1=0{\cdot}02wl-\frac{ql}{2},\quad R_2=\frac{ql}{2},\text{ and } q=0{\cdot}18w.$$

Therefore at the same point C we have

$$M_f=-0{\cdot}02wly+\frac{qly}{2}-\frac{qy^2}{2}=wy(0{\cdot}07l-0{\cdot}09y).$$

$$\therefore M_f \text{ at } C=\frac{wy}{4}(l-y)+wy(0{\cdot}07l-0{\cdot}09y)=wy(0{\cdot}32l-0{\cdot}34y)\ldots.[A].$$

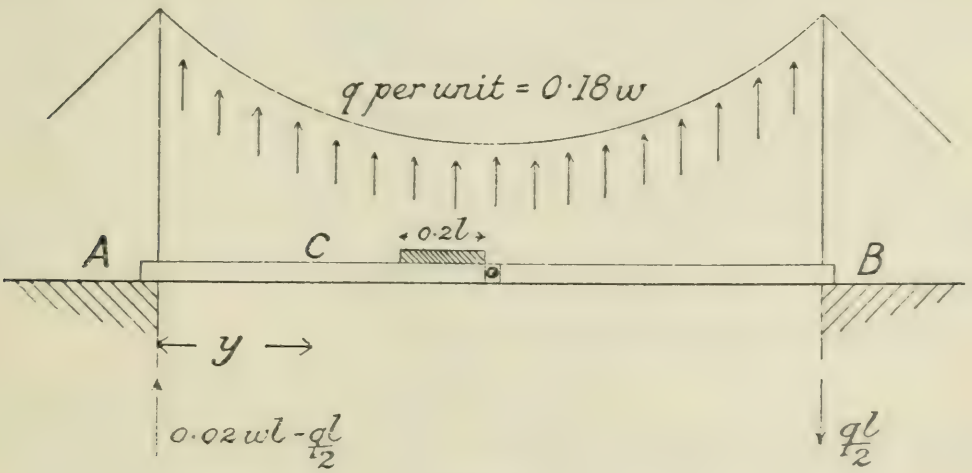

FIG 17.

For a maximum $\frac{dM_{ff}}{dy} = 0{\cdot}32w l - 0{\cdot}68wy = 0.$

$$\therefore \quad y = \frac{8}{17} l,$$

and $$M_{ff} = +0{\cdot}0753 w l^2 \ldots\ldots\ldots\ldots\ldots\ldots [B].$$

The maximum sagging bending moment at C will be caused by the load on the remainder of the span, *Fig.* 18. Here we have

$$R_1 = 0{\cdot}48wl - \frac{ql}{2}, \quad R_2 = \frac{ql}{2}, \text{ and } q = 0{\cdot}32w.$$

At the point C, distant y from A, we have

$$M_f = -0{\cdot}48wly + \frac{qly}{2} - \frac{qy^2}{2} + \frac{wy^2}{2}$$

$$= wy\,(-0{\cdot}32l + 0{\cdot}34y) \ldots\ldots\ldots\ldots\ldots\ldots [C].$$

This is the same as equation [A] with the sign reversed.

$$\therefore \quad M_{ff} = -0{\cdot}0753 w l^2 \ldots\ldots\ldots\ldots\ldots\ldots [D].$$

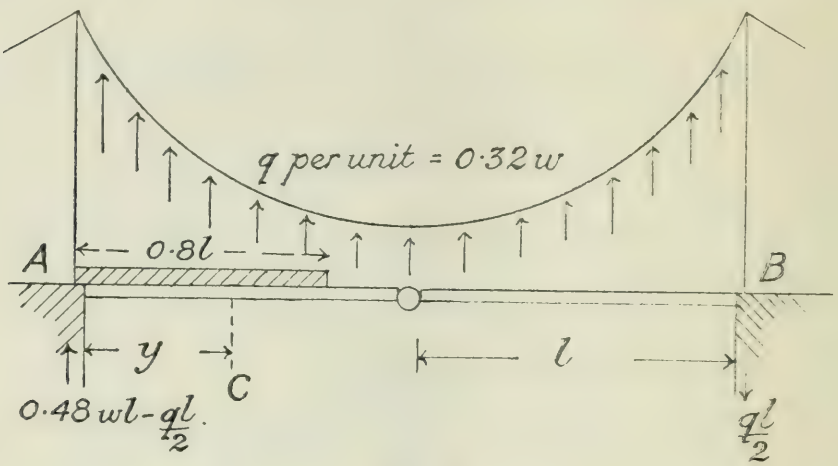

FIG. 18.

It can be ascertained easily by differentiation that for any given length of the load, as in *Fig.* 18, measured from A towards the centre, say x, that the point of M_{ff} is given by the value of $y = \frac{lx(4l - 3x)}{2(2l^2 - x^2)}$, and that the equation $8l^3 - 12l^2x + 3x^3 = 0$ gives the length of load that causes the greatest possible sagging M_f at any point on the left half span. This gives a value of $0{\cdot}79l$ to x, which makes $y = 0{\cdot}468l$, and $M_{ff} = \pm 0{\cdot}07535 w l^2$ .....[E].

Giving w the value of 25 lbs., we get

$$M_{ff} = \pm 1{\cdot}884 l^2 \ldots\ldots\ldots\ldots\ldots\ldots [F],$$

which compares most favourably with the $M_{ff} = -3{\cdot}184 l^2$ (equation [75]) obtained in the previous case, and shows how great is the

economy effected by the central hinge. This $M_{\prime\prime}$ is equivalent to a maximum stress of

$\pm 3{\cdot}03$ tons per square inch in the girder flanges .....[G].

It is obvious that if the hinge is constructed so as to act horizontally (as well as vertically), no stresses can be caused in the flanges of the horizontal girder by wind pressure; and we have obtained the maximum stress in the flanges under any conditions.

MAXIMUM TENSION IN MAIN CABLES.

The maximum horizontal tension in the main cables, under a permanent load p, and a moving load w, per unit length is of course

$$\frac{(p+w)l^2}{2h} = \frac{45 \times 1200^2}{400 \times 2240} = 72{\cdot}3 \text{ tons.}$$

At the standards the maximum tension is $72{\cdot}3 \sec\phi$

$= 72{\cdot}3 \times 1{\cdot}054 = 76{\cdot}20$ tons $= 9{\cdot}52$ *tons per square inch*...[H].

In the previous example, equation [77], the maximum tension was 9·49 tons per square inch; as in that case although there was a temperature load, the stiffening girders took ·07 of the full moving loads.

This slight increase in tension in the cables is insignificant compared with the great reduction in girder stresses.

The maximum deflection in the main cables due to the permanent and moving loads will be

$$\frac{(20+25)l^4}{2h^2CE} = \frac{45 \times 1200^4}{2 \times 200^2 \times 8 \times 30{,}000{,}000} = 4{\cdot}86'.$$

The maximum deflection due to temperature will be (equation [4], Chapter II.),

$$\pm \frac{\Delta l^2}{h} = \pm \frac{12000^2}{3000 \times 200} = \pm 2{\cdot}40''.$$

MAXIMUM TENSION IN WIND CABLES.

The maximum horizontal tension in the wind cables will be

$$\frac{5l^2}{2h} = \frac{5 \times 1200^2}{2 \times 300} = 12{,}000 \text{ lbs.} = 5{\cdot}36 \text{ tons.}$$

At the abutments the tension is $5{\cdot}36 \sec\phi$

$= 5{\cdot}36 \times 1{\cdot}118 = 5{\cdot}95$ tons .....[I];

which is also the tension per square inch.

It is obvious that there is no temperature stress in the wind cables, as the central hinge will permit of lateral movement, and the wind guys need not be kept tight artificially.

SHEARING STRESSES.

The permanent, or any equally distributed load, cannot cause any stresses in the girders on account of the central hinges.

Referring back to *Fig.* 15, we see that as regards the *unloaded* half span, the shearing force at the abutment A is equal to the shearing force at the central hinge, but of opposite sign ; while at the centre of the unloaded span the shearing force is always zero. Therefore at A we have

$$S = -R_1 = -\frac{ql}{2};$$

and the bigger q is the bigger S is ;

$\therefore$ for an *unloaded* half span, a full load on the other half span gives the largest shearing force ; *vide Fig.* 16, *i.e.*,

at A, $$-S \text{ (max.)} = -\frac{ql}{2} = -\frac{wl}{4},$$

at O, $$+S \text{ (max.)} = +\frac{ql}{2} = +\frac{wl}{4}.$$

Next consider the abutment B, of a loaded span ; (*Fig.* 15). Here

$$+S = +R_2 = +\frac{w(3x-l)}{2l},$$

i.e., S is positive so long as x is $> \frac{l}{3}$; or in other words a load, extending from the centre towards B for a distance of $\frac{l}{3}$, causes a negative shearing stress at B ; this agrees with Table IV. *Fig.* 19 therefore gives the maximum negative shearing stress at A.

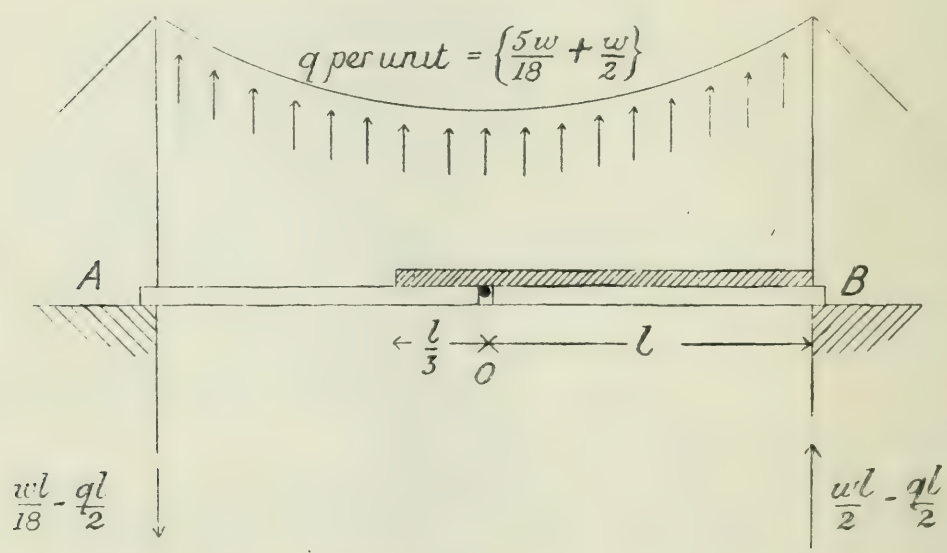

FIG. 19.

$$\therefore \text{ at A, } \quad -S\,(\text{max.}) = R_1 = \frac{wl}{18} - \frac{ql}{2} = -\frac{wl}{3}.$$

The reverse loading, *i.e.*, from A for a distance of $\frac{2l}{3}$ towards O gives the positive maximum value to S, at A.

In this case

$$+S\,(\text{max.}) = R_1 = \frac{4wl}{9} - \frac{ql}{2} = \frac{4wl}{9} - \frac{wl}{9} = +\frac{wl}{3}.$$

$$\therefore \quad S\,(\text{max.}) = \pm\frac{wl}{3} = \pm 4{\cdot}46 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots [J].$$

The greatest stress in the vertical bracing will be

$$+\frac{4{\cdot}46}{2} \text{ tons (compression)} \ldots\ldots\ldots\ldots\ldots\ldots [K]$$

and in the diagonals

$$-\frac{4{\cdot}46}{2}\sqrt{2} = -3{\cdot}15 \text{ tons (tension)} \ldots\ldots\ldots\ldots\ldots\ldots [L].$$

The wind pressure will cause only a direct tension in the short braces of the horizontal girder, as the central hinges prevent all bending moments and shearing stresses, under symmetrical loads.

This direct tension on a 15-foot panel

$= 180 \times 5 = 900$ lbs., say 0·35 tons, an insignificant amount... [M].

In the previous example, Chapter II., we obtained maximum shearing stresses of $+5{\cdot}21$ and $-4{\cdot}27$ tons for the stiffening girders, and $\pm 1{\cdot}5$ tons for the wind girder.

We see therefore that the introduction of central hinges almost halves the M_{ff}, and reduces the maximum shearing stresses in the girders; while the effect upon the wind guys is to eliminate the temperature stresses, which in Case I., equation [54], amount to 8·93 tons per square inch; and in the wind girder the shearing stresses are eliminated also.

CHAPTER IV.

SUSPENSION BRIDGE WITH HANGING GIRDERS.

IN this case the girders are not hinged, nor do they rest on the abutments. They hang suspended from the main cables, which carry all the load, permanent and moving.

MAXIMUM TENSION IN MAIN CABLES.

It is clear that the maximum horizontal tension in the main cables is

$$\frac{(p+w)l^2}{2h} = \frac{45 \times 1200^2}{2 \times 200 \times 2240} = 72{\cdot}3 \text{ tons};$$

and the maximum tension at the standards $= 72{\cdot}3 \sec\phi = 76{\cdot}2$ tons, $= 9{\cdot}52$ tons per square inch; the same result as in Case II., Chapter III., of the hinged girder, equation [H]; but this will be reduced a little on account of the stiffness of the girders throwing a portion of every symmetrical load outwards, as explained below; and the maximum tension will be, as in Case I., Chapter II., viz.:— 9·49 tons per square inch.

TEMPERATURE STRESSES IN GIRDERS.

If the stiffening girders lie horizontal at the mean temperature, the permanent load causes no stresses in them. If however the temperature falls, the centre of the cables rise, lifting the centres of the girders also. The ends of the girders however remain at the same level; any shortening in the end vertical suspenders being cancelled by an equal shortening in the standards. The girder by its stiffness transfers a portion of the load outwards, away from the centre, and the stresses set up are therefore practically the same as in Case I., Chapter II.; for the shape assumed by the girder is the same, and the maximum temperature stresses in the girder flanges are $\pm 1{\cdot}25$ tons per square inch, *vide* page 11, equation [14] ....[N].

MOVING LOAD STRESSES IN GIRDERS.

As stated above, the vertical suspension rods are so adjusted that the girders lie horizontal at the mean temperature under the permanent load; and apart from temperature stresses (and wind stresses, which will be dealt with below) only the moving load affects the girders.

Now the girders are kept in equilibrium by two sets of forces only, (i.) the weights on it acting vertically downward, (ii.) the tensions in the suspension rods acting vertically upwards. These two sets of forces must be numerically equal and opposite, and the centre of gravity of all the loads on the girders must act in the same vertical line as the centre of force of all the tensions acting upwards.

It is obvious therefore that the tensions in the vertical rods are no longer equal, and that the girders cannot distribute uniformly on to the cables an unsymmetrical load; and that consequently the cables must undergo some distortion. This system therefore fails in its object from the beginning; the girders merely form a sort of stiffened roadway, and if the moving load were very large compared to the permanent load the distortion would be so great that the ends of the roadway might rise in the air under certain conditions. In an *unstiffened* suspension bridge the roadway would merely "snake" under the same conditions.

This state of affairs however is unlikely, and as we have taken $p : w = 20 : 25$, we can assume that under all conditions there will be some tension set up in every vertical rod.

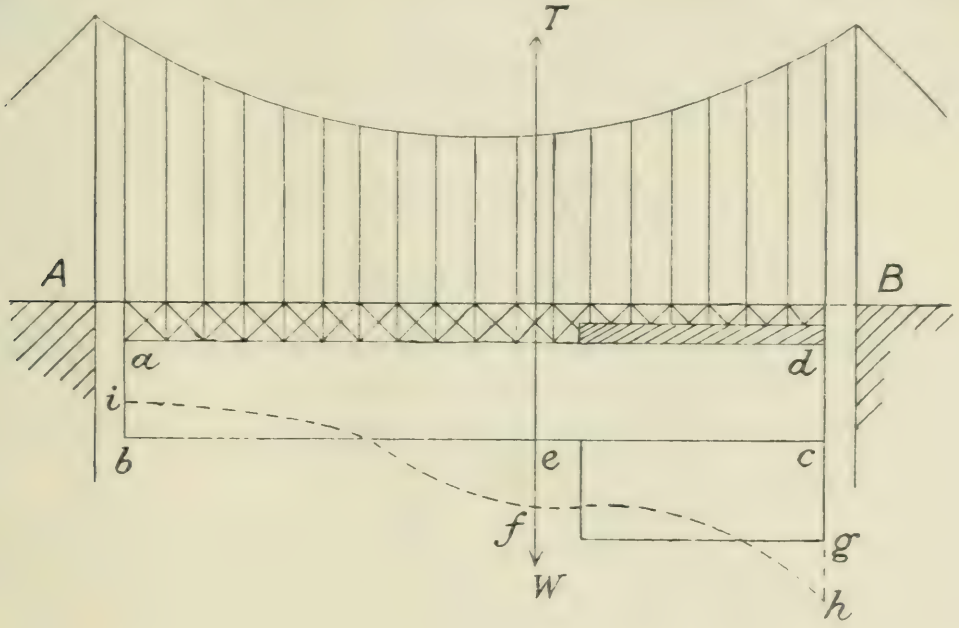

FIG. 20.

Now in *Fig.* 20 the permanent load on the girders can be represented by the rectangle *abcd*, where $ab = cd = p$, the weight per unit length.

If a moving load be upon the bridge as shown, it can be represented by the rectangle *cefg*, where $cg = ef = w$, the weight per unit length. The centre of gravity of these loads acts at the centre of gravity of the rectangles, and is shown by the arrow marked W.

Now the tensions in the vertical rods *must* be represented by some such area as *adhi*, which is equivalent in area to the sum of the two

rectangles, *abcd* and *cefg*; and of which the centre of gravity is in the same straight line as W, and is shown by the arrow marked T, acting upwards.

T = W, and the ordinates of *adhi* give the varying tensions in the vertical rods.

From this we see that certain hogging bending moments are set up in the girders by the moving load, but that they are small; and since the girders are not *anchored* at the abutments it follows that the maximum hogging M_f *must be less* than that found in Case I., Chapter II.

We can neglect therefore the hogging bending moments and deal only with the sagging bending moments.

The tensions in the vertical rods correspond much more closely with the loads immediately below them than in the previous cases.

The deflection of the girders will be greatest when the cables are deflected most; and this will occur under the maximum moving load.

As before the girders will transfer outwards from the centre a portion of this symmetrical load, thus reducing the deflection of the cables, and the tension in them; and as the two outer ends of the girders do not change their position vertically, the net result is almost the same as that in Chapter II., equations [19], [20], and [21], as the girders assume the same shape under the load.

The maximum flange stress in the girders will be

1·4 tons per square inch .....[O].

WIND STRESSES.

To this must be added the stresses caused in the bottom flanges by the wind pressure acting on the horizontal girder.

The deflection of the wind cable at the centre due to a wind load of 5 lbs. per inch-run $= \frac{3wl^4}{8h^2CE}$, *vide* [45].

In this case the cable takes the whole of the load, as the girders do not rest on the abutments.

Now if we equate the deflections of the cable and the girder we get $\frac{3wl^4}{8h^2CE} = \frac{5ql^4}{24EI}$, where w is the wind pressure per unit length, and q the equivalent load on the girder.

$$\therefore \quad \frac{3 \times 5 \times l^4}{8 \times 300^2 \times 1 \times E} = \frac{5ql^4}{6Gd^2E} = \frac{5ql^4}{6 \times 8 \times 180^2 \times E},$$

whence $q = 6{\cdot}48$ lbs. per inch-run. This is more than the wind pressure, and consequently we see that the inherent stiffness of the horizontal girder transfers part of the load towards its outward ends and prevents the cable deflecting so much at the centre as it would i.

the girder had no stiffness whatever; or in other words the tensions in the tie-rods are greater near the abutments than near the centre. The maximum tension in the wind cable would be reduced a little, and the stress in the girder would be much less than that which has been deduced from the deflection of the cable in the preceding paragraph.

Fig. 21 represents the state of affairs approximately; *abcd* represents the load of 5 lbs. per inch, and *aegfd* represents the tensions in the rods.

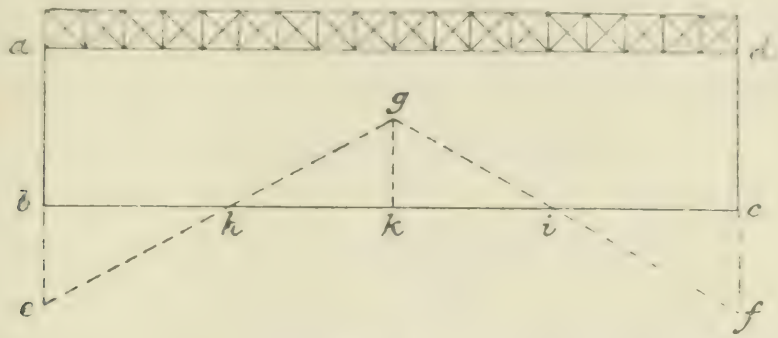

FIG. 21.

The area $aegfd = abcd$.

$\therefore$ $ebh = hgk$, and $bh = hk = \frac{l}{2}$.

$\therefore$ the M_f at the centre of the girder

$$= \left(ebh \times \frac{5l}{6}\right) - \left(hgk \times \frac{l}{6}\right) = ebh \times \frac{2l}{3}.$$

Now $$ebh = \frac{eb}{2} \times \frac{l}{2},$$

$\therefore$ $$M_{ff} = eb \times \frac{l^2}{6} = M_r = \frac{rGd}{2},$$

$\therefore$ $$\frac{eb \times 1200^2}{6} = \frac{r \times 8 \times 180}{2},$$

whence $$eb = \frac{3r}{1000}.$$

The exact value of eb is indeterminate, since we do not know the deflection of the wind cable, but it cannot be more than 5, and is probably less.

Assuming this extreme value, $r = \frac{5 \times 1000}{3}$ lbs. $= 0{\cdot}74$ tons per square inch.

Therefore the maximum flange stress in the girders

$$= 1{\cdot}25 + 1{\cdot}40 + 0{\cdot}74 = 3{\cdot}39 \text{ tons per square inch} \ldots\ldots [P].$$

SHEARING STRESSES.

The maximum shearing stresses in the *stiffening* girders can be deduced approximately from the maximum bending moment. The maximum bending moment caused by a uniform load of w per unit on a span of $2l$ is

$$\frac{wl^2}{2}, \text{ which} = M_r = \frac{2rI}{d} = \frac{rGd}{2}.$$

Now the maximum flange stress in the stiffening girders due to temperature and moving load is $1{\cdot}40 + 1{\cdot}25 = 2{\cdot}65$ tons per square inch. [The shearing stresses caused by the wind pressure are taken by the braces in the horizontal girder.]

$$\therefore \quad \frac{wl^2}{2} = \frac{rGd}{2} = \frac{2{\cdot}65 \times 16 \times 50}{2},$$

$$\therefore \quad wl = \frac{2{\cdot}65 \times 16 \times 50}{1200} = 1{\cdot}77 \text{ tons.}$$

Now the shearing force at the abutments, under the conditions stated, $= wl$.

$$\therefore \quad S = 1{\cdot}77 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots [Q].$$

$\therefore$ the tension in the vertical braces

$$= \frac{1{\cdot}77}{2} = +0{\cdot}88 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots [R],$$

and the compression in the diagonal cross-braces

$$= -0{\cdot}88\sqrt{2} = -1{\cdot}24 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots [S].$$

For the wind girder $r = 0{\cdot}74$ tons.

$$S = wl = \frac{rGd}{l} = \frac{0{\cdot}74 \times 8 \times 180}{1200} = 0{\cdot}89 \text{ tons} \ldots\ldots\ldots\ldots [T].$$

$\therefore$ the tension in the cross-braces

$$= +0{\cdot}89 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots [U],$$

and the compression in the diagonals

$$= -0{\cdot}89\sqrt{2} = -1{\cdot}26 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots [V].$$

MAXIMUM TENSION IN WIND CABLES.

The maximum tension in the wind cables will be the same as in Case II., Equation [I], viz. :—

$$5{\cdot}95 \text{ tons per square inch} \ldots\ldots\ldots\ldots\ldots\ldots [W],$$

if we neglect the diminution due to the altered disposition of the wind load effected by the stiffness of the wind girder.

VERTICAL TIE-RODS.

It will be noted that in this case the maximum tension in the vertical suspension rods is greater than in the previous instances, *vide Fig.* 20. If the rods are 50′ apart, the normal maximum tension is $(p+w)\times 50=45\times 50=2{,}250$ lbs., say 1 ton; in this case we must allow for a maximum of about 1·5 tons in the rods near the abutments. In the same way the tension in the tie-rods of the wind cable will be greater than in other cases. Assuming them to be 180″ apart, the maximum tension in the rods near the abutments will be about $2\times(180\times 5)=1{,}800$ lbs.$=0{\cdot}8$ ton.

CHAPTER V.

RECAPITULATION.

Conditions.—Span, 200′ ; roadway, 15′ wide over all ; dip of main cables, one-twelfth of the span, =200″ ; total cross-sectional area of main cables, 8 square inches ; back guys, sloping at about 5 6.

Load.—240 lbs. per foot-run permanent ; 300 lbs. per foot-run moving ; 60 lbs. per foot-run wind pressure.

Stiffening Girders.—Depth, 4′ 2″ ; total cross-sectional area of booms, 16 square inches ; vertical and cross-diagonal braces ; square panels.

Wind Cables.—Depth of parabola, one-eighth of span, =300″ ; cross-sectional area, 1 square inch each.

Wind Girder.—Depth, 15′ ; square panels ; cross-braced diagonals and cross braces.

TABLE V.

	Continuous Girders.	Hinged Girders.	Hanging Girders.	
Maximum tension in main cables	9·49	9·52	9·49	tons per sq. in.
Maximum flange stress in stiffening girders ...	5·12	3·03	3·39	,, ,,
Maximum shearing stress in vertical girders ...	5·21	4·46	1·77	total stress.
Maximum tension in wind cables	12·62	5·95	5·95	tons per sq. in.
Maximum shearing stress in wind girder	1·5	0·35	0·89	total stress.

CHAPTER VI.

UNBRACED TRUSSED BEAMS.

IF an unbraced trussed beam carry an unsymmetrical load, it is clear that the same principles (which have been discussed in Chapter II.) hold good, if there is continuity of fibre in the upper beam. The stiffness of the upper beam saves the structure from collapse, by transferring a portion of the load on to each strut, and by carrying a portion of the load by its own stiffness.

If the points of the trussing lie on a parabola, the load on every strut will be the same, and the structure may be likened to a stiffened suspension bridge, in which the roadway is carried on the top of the cables by columns, instead of being suspended from it by rods.

Let *Fig.* 1 represent an unbraced trussed beam carrying a concentrated load over one strut, at B.

The loaded strut must sink a little, the tie-rod is thereby elongated, and the unloaded strut tends to be forced upwards; the far end of the beam, A^1, being kept down by the pull of the tie-rod.

Now (within the elastic limit) any such deformation of the truss must be very small; and therefore the tensions in the two sloping tie-rods are practically the same, since the centre tie-rod is assumed to remain horizontal. In consequence the compression in each strut is the same, and therefore the beam transfers an equal portion of the load $\left(\frac{W}{n}\right)$ to the other strut at B^1, and carries a portion itself. The state of affairs is illustrated in *Figs.* 1, 2, and 3 (see page 44). *Fig.* 1 represents the truss as a whole; *Fig.* 2 represents that portion of the load taken up by the truss system; *Fig.* 3 represents the loading of the beam; the compressions in the struts pressing it up at B and B^1, the vertical components of the tensions in the tie-rod holding it down at A and A^1, while the load W at B causes the usual reactions at A and A^1.

The trussing of course causes a direct compression in the beam as well.

The value of n is found by equating the deflections at the centre of the two systems, *Figs.* 2 and 3.

Now the deflection of the *truss*, *Fig.* 2, is made up of three parts:

(i.). That due to the stretching of the tie-rod.
(ii.). That due to the compression of the upper beam.
(iii.). That due to the shortening of the struts.

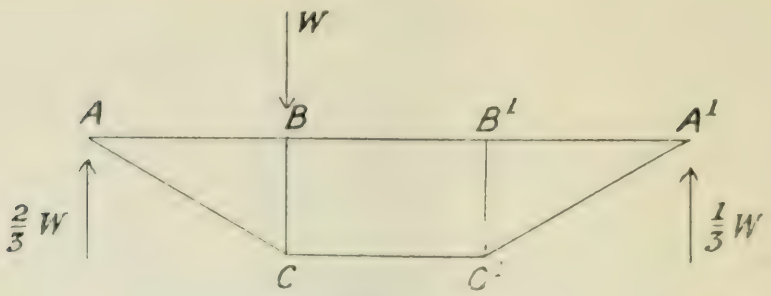

FIG. 1.

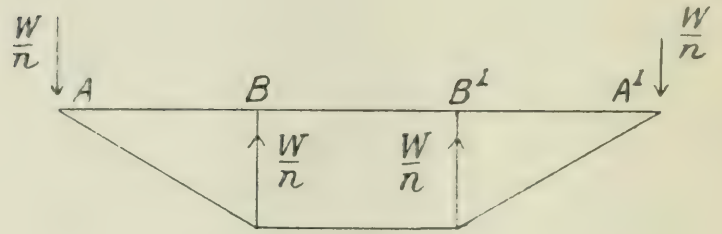

FIG. 2.

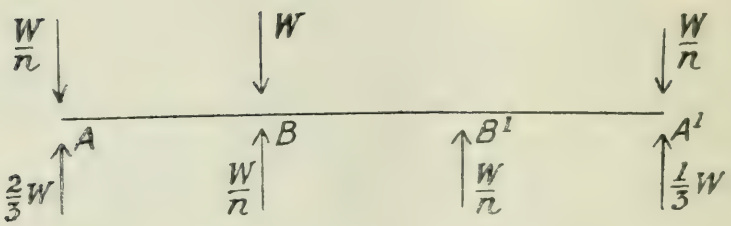

FIG. 3.

Now the struts are very short, and the stress per square inch in them is very small; therefore their compression is infinitesimal; and it can be neglected without perceptible error. It will be allowed for by taking the tension in the tie as uniform throughout, and equal to its maximum value; whereas the tension in the centre rod is less than that in the sloping rods. The infinitesimal errors, occasioned by neglecting both, practically cancel each other.

Next we may consider the truss as a parabolic girder, with only two vertical rods, the upper boom being horizontal and the lower boom being the chords of the parabola. From Chapter II., equation [A], we know that the deflection due to the stretching of the tie-rod or cable is $\frac{3\hat{c}l^2}{4h}$; and we must ascertain the additional deflection caused by the shortening of the upper boom.

The length of the arc of the parabola is $2l\left(1+\frac{2h^2}{3l^2}\right)$, as we have seen; and the span $2l$ now becomes $2l(1-\Delta)$, with a corresponding dip at the centre of h_1 instead of h.

$$\therefore \quad 2l\left(1+\frac{2h^2}{3l^2}\right)=2l\,(1-\Delta)\left\{1+\frac{2h_1^2}{3l^2\,(1-\Delta)^2}\right\},$$

for the length of the arc is assumed to be unaltered,

whence $$h_1=h+\frac{3\,\Delta\,l^2}{4h}, \text{ approx.}$$

$\therefore$ the deflection $$=h_1-h=\frac{3\,\Delta\,l^2}{4h}$$

and the total deflection $$=\frac{3l^2}{4h}(\delta+\Delta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots[1].$$

As in this instance the tie-rod is of steel, and the upper beam of wood, the values of E are very different; being in the proportion of 20 to 1; viz., 30,000,000 lbs. for steel, (E_1), and 1,500,000 for the timber, (E_2).

$$\delta=\frac{\text{tension per square inch}}{\text{modulus of elasticity}}=\frac{\text{total tension}}{\text{area}\times E_1} \quad \ldots\ldots\ldots\ldots[2].$$

Similarly,

$$\Delta=\frac{\text{compression per square inch}}{\text{modulus of elasticity}}=\frac{\text{total compression}}{\text{area}\times E_2} \quad \ldots[3].$$

In the following examples the same trussed beam will be considered for purposes of comparison, the dimensions *between centres* being as follows :—

Length of upper beam 36′, trussed at equal thirds; the size of the beam being $4\frac{1}{2}''\times 9''$, and the beam being continuous.

Length of struts, 5′; dimensions, $4\frac{1}{2}''\times 4\frac{1}{2}''$.

Tie-rod, 1″ diameter steel.

The value of E for steel is 30,000,000 lbs., and for the timber 1,500,000 lbs.

$$\therefore \quad E_1 : E_2 = 20 : 1.$$

The value of I for the beam is

$$\frac{1}{12}\times\frac{9}{2}\times\frac{10^3}{1}=375.$$

The upper beam will be assumed to be supported laterally at the struts by cross-rafters, consequently

$$\frac{l}{d}=\frac{144}{10}=14{\cdot}4,$$

and the maximum safe compression will be taken at 800 lbs. per square inch.

$$AB=BB^1=B^1A=12'=CC^1.$$
$$BC=B^1C^1=5'.$$
$$AC=C^1A^1=13'.$$

Case I.—Concentrated load over one strut (*Figs.* 1, 2, and 3).

The direct compression in the beam (*Fig.* 2) is $\frac{12W}{5n}$, and the tension in AC or A^1C^1 is $\frac{13W}{5n}$, for

$$AB : BC : CA = 12 : 5 : 13.$$

The compression in each strut is $\frac{W}{n}$.

The upward deflection of the beam at its centre, due to the two pressures of the struts, is given by the equation

$$v_1 = \frac{W}{6nE_2I}\{2l^3 - 3lx^2 + x^3\},$$

x being equal to $\frac{l}{3}$, and the span $= 2l$.

The downward deflection of the beam at its centre, due to the load W,

$$= v_2 = \frac{W}{12E_2I}\{2l^3 - 3lx^2 + x^3\},\ x \text{ being equal to } \frac{l}{3}.$$

$\therefore$ The net downward deflection

$$= v_2 - v_1 = \frac{W}{6E_2I}\{2l^3 - 3lx^2 + x^3\}\left(\frac{1}{2} - \frac{1}{n}\right) \quad \ldots\ldots\ldots\ldots [4].$$

The deflection of the truss at its centre $= \frac{3l^2}{4h}(\delta + \Delta)$, where l = the half span, 216″; h = the dip, 60″;

$$\delta = \frac{\text{tension in tie}}{\text{area} \times E_1} = \frac{13W}{5n \times 1 \times E_1} = \frac{13W}{5n \times 20E_2},\ \text{(since } E_1 = 20E_2\text{)}.$$

$$\Delta = \frac{\text{compression in beam}}{\text{area} \times E_2} = \frac{12W}{5n \times 45 \times E_2}.$$

$$\therefore \qquad v = \frac{3l^2}{4h}(\delta + \Delta) = \frac{11Wl^2}{4{,}800 \times E_1 \times n} \quad \ldots\ldots\ldots\ldots [5].$$

Equating [4] and [5], and giving x its value of $\frac{l}{3}$, we get

$$\frac{23Wl^3(n-2)}{162 \times E_2 \times I \times n} = \frac{11Wl^2}{4{,}800 \times E_2 \times n};\quad l = 216'',\ I = 375,$$

$$\therefore \qquad n = 2{\cdot}028 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6],$$

and we see that very nearly one half of W is transferred to the unloaded strut at B^1.

The compression in each strut is $\frac{W}{2{\cdot}028}$ tons.

The tension in the tie is $\frac{13W}{10{\cdot}14}$ tons.

The direct compression in the beam is $\frac{12W}{10{\cdot}14}$ tons.

The sagging M_{π} at $B = \left(\frac{W}{n} - \frac{2W}{3}\right) 12 = \frac{4W(3-2n)}{n} = -\frac{4{\cdot}224W}{2{\cdot}028}$.

The hogging M_{π} at $B^1 = \left(\frac{W}{n} - \frac{W}{3}\right) 12 = \frac{4W(3-n)}{n} = +\frac{3{\cdot}888W}{2{\cdot}028}$.

The absolute M_{π} in this case is the sagging one at B.

If W equals one ton, $M_{\pi} = -\frac{4{\cdot}224}{2{\cdot}028}$ foot-tons $= M_r$.

$$\therefore \qquad M_r = \frac{r \times 450}{6} = \frac{4{\cdot}224}{2{\cdot}028} \times 12 \times 2240,$$

(units being inches and lbs.) whence $r = 746$ lbs. per square inch.

The direct compression $= \frac{12}{10{\cdot}14} \times \frac{2240}{45} = 59$ lbs. per square inch.

With a value of one ton for W, we get the following maximum stresses :—

In the upper beam, $746 + 59 = 805$ lbs. per sq. in. compression.
In the struts, (area $4\frac{1}{2}'' \times 4\frac{1}{2}''$), 55 lbs. per sq. in. compression.
In the tie-rod, 1·282 tons per sq. in. tension.

TEMPERATURE STRESS.

The next point to be considered is temperature stress. The upper beam, being of timber, will not be affected ; but the tie-rod being of steel will elongate and contract.

Taking a range of temperature equal to $\pm 50^{\circ}$ F., as in (Chapter II.), the deflection at the centre of the truss, due to this cause, is

$$\frac{3\triangle l^2}{4h}\text{ ; where } \triangle = \frac{50}{150{,}000} = \frac{1}{3{,}000}.$$

If the temperature falls, the tie-rod shortens, and the struts are compressed ; if the temperature rises, the reverse action takes place. Let Q be the pressure in each strut, when the temperature falls 50° F.

The deflection upward of the beam at its centre, due to these two pressures,

$$= \frac{Q}{6E_2I}\left\{2l^3 - 3lx^2 + x^3\right\} = \frac{23Ql^3}{81E_2I}, \text{ since } x = \frac{l}{3}.$$

*This must equal $\frac{3\triangle l^2}{4h}$ or $\frac{l^2}{4{,}000h}$.

* Strictly the net upward deflection of the tie-rod is $\frac{3\triangle l^2}{4h}$ minus the downward deflection due to the pressures, Q, in the struts, viz., $-\frac{39Ql^2}{20hE_1}$, but the error caused by neglecting this is inappreciable.

$$\therefore \quad \frac{23Q/^3}{81E_2I} = \frac{/^2}{4{,}000}.$$

$/ = 216''$, $h = 60'$, $E_2 = 670$ tons for timber, $I = 375$.

$$\therefore \quad Q = \cdot 017 \text{ ton} = 38 \text{ lbs.}$$

This is the increased pressure in each strut.

The increased hogging M_f at $B^1 = 38 \times 12 \times 12 = 5{,}472$ inch-lbs., equivalent to an increased stress of 73 lbs. per square inch.

The increased direct compression in the beam is $\frac{38 \times 12}{5} = 91$ lbs., equivalent to 2 lbs. per square inch.

∴ the total maximum temperature stress in the beam is 75 lbs. per square inch.

The increased tension in the tie-rod

$$= \frac{0{\cdot}017 \times 13}{5} = 0{\cdot}044 \text{ tons.}$$

If the temperature rises 50° F., the reverse action takes place ; the tension in the tie-rod is reduced by 0·044 tons, the hogging M_f at B^1 is reduced by 5,472 inch-lbs., and the sagging M_f at B is increased by that amount.

We get therefore the following maximum stresses :—

In the upper beam, $805 + 75 = 880$ lbs. per sq. in. compression... [7].
In the struts, $55 + \frac{1}{2} = 56$ lbs. per sq. in. compression ..... [8].
In the tie-rod, $1{\cdot}282 + 0{\cdot}044 = 1{\cdot}326$ tons per sq. in. tension... [9].

Case II.—Symmetrical concentrated loads over each strut.

In this case the value of n must be half the amount obtained in equation [6], since the compression in each strut is exactly doubled, and equals $\frac{2W}{2{\cdot}028}$;

$$\therefore \quad n = 1{\cdot}014 \quad \ldots\ldots\ldots\ldots\ldots\ldots [10].$$

The compression in each strut is $\frac{W}{1{\cdot}014}$ tons.

The tension in the tie-rod is $\frac{13W}{5{\cdot}07}$ tons.

The direct compression in the beam is $\frac{12W}{5{\cdot}07}$ tons.

The M_{ff} is a sagging one, constant between B and B^1, and

$$= \left(\frac{W}{n} - W\right)12 = -\frac{0{\cdot}168W}{1{\cdot}014} \text{ foot-tons.}$$

The resulting stress is due almost entirely to the great disparity in values of E_1 and E_2, for steel and timber, and to the large value of I for the upper beam ; and as we shall see cannot be neglected.

This shows how uneconomical composite structures are, as even with symmetrical loading appreciable bending moments are set up, if

there is continuity of fibre beyond the panel points. Let the load on each strut be 1 ton.

$M_{ff}=4{,}453$ inch-lbs., and the resulting value of $r=59$ lbs. per sq. in.

The direct compression in the upper beam $=118$ lbs. per sq. in.

We get therefore the following maximum stresses for a load of 1 ton on each strut :—

In the upper beam, 177 lbs. per sq. in. compression.

In the struts, 110 lbs. per sq. in. compression.

In the tie-rod, 2·564 tons per sq. in. tension.

A load of *4 tons* on each strut will multiply these stresses by four ; and the temperature stresses will be the same as in the previous case.

Therefore for a load of 4 tons on each strut, we have

In the upper beam, $708+75=785$ lbs. per sq. in. compression...[11].
In the struts, $440+\frac{1}{2}=441$ lbs. per sq. in. compression .....[12].
In the tie-rod, $10{\cdot}256+0{\cdot}044=10{\cdot}30$ tons per sq. in. tension...[13].

Case III.—Distributed loads.

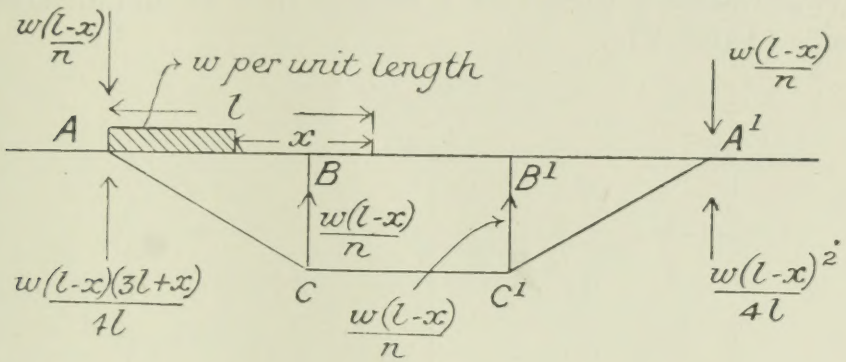

FIG. 4.

Let *Fig.* 4 represent a uniformly distributed load over part of the beam ; x being measured from the centre of the beam. The compressions in the struts, etc., will be as shown.

The upward deflection of the beam at its centre, due to the pressures in the struts,

$$=v_1=\frac{w(l-x)}{6nE_2I}\left\{2l^3-3l\left(\frac{l}{3}\right)^2+\left(\frac{l}{3}\right)^3\right\}=\frac{23wl^3\,(l-x)}{81nE_2I}.$$

The downward deflection of the beam at its centre, due to the load of w per unit,

$$=v_2=\frac{w}{12E_2I}\{2l^3(l-x)-l(l^3-x^3)+\tfrac{1}{4}(l^4-x^4)\}.$$

The net downward deflection of the beam therefore, v,

$$=v_2-v_1=\frac{w(l-x)}{3E_2I}\left\{\frac{5l^3-3l^2x-3lx^2+x^3}{16}-\frac{23l^3}{27n}\right\}.....[14].$$

The deflection of the truss at its centre

$$= \frac{3l^2}{4h}(\delta + \Delta),$$

$$\delta = \frac{13w(l-x)}{5n \times E_1} = \frac{13w(l-x)}{5n \times 20E_2}, \text{ since } E_1 = 20E_2.$$

$$\Delta = \frac{12w(l-x)}{5n \times A \times E_2} = \frac{4w(l-x)}{5n \times 15 \times E_2}, \text{ since } A = 45 \text{ sq. in.}$$

$$\therefore \quad v = \frac{3l^2}{4h}(\delta + \Delta) = \frac{11wl^2(l-x)}{4{,}800 \times E_2 \times n}, \text{ since } h = 60'' \quad \ldots\ldots\ldots[15].$$

Equating [14] and [15], and putting I = 375, we get

$$\frac{1}{15}\left\{\frac{5l^3 - 3l^2x - 3lx^2 + x^3}{16} - \frac{23l^3}{27n}\right\} = \frac{11l^2}{64n}$$

or
$$27n\,(5l^3 - 3l^2x - 3lx^2 + x^3) - 368l^3 = \frac{4455l^2}{4} \quad \ldots\ldots\ldots[16].$$

By giving different values to x from 0 to l we obtain successive values for n (Table VI.).

TABLE VI.

$\frac{x}{l}$	n
0	2·76
0·1	2·96
0·2	3·22
0·3	3·58
0·4	4·08
0·5	4·81
0·6	5·92
0·7	7·79
0·8	11·59
0·9	23·07

Note.—The compression in the struts of the truss under any irregular distribution of loading can be calculated from this Table.